Lecture 1

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Welcome to the 305

Operations Research (OR)

- Many different names (Management Science, Business Analytics, etc.)
- Discipline dealing with the application of advanced analytical methods to help make better decisions
- Typically, there is a quantity that needs to be optimized
- The objective function is a mathematical expression describing the quantity that we want to analyze in terms of key variables
- Goal is to find best value of variables that either maximizes or minimizes the objective function

Models in OR

- A model is a mathematical representation of a problem using mathematical relationships involving key variables, the objective function, and constraints

- Two types of OR models
  - Deterministic models assume the relationships of a problem are fully known
  - Stochastic models allow some relationships to contain randomness

- First part of course is focused on deterministic models

Ex: Production of Steel

- Crysteel is a company that makes and sells steel products
- Cost of $5 to produce one unit and each unit sells for $20
- Each unit requires 4lbs. of steel and Crysteel has 100lbs. of steel
- Q: How many units should be produced to maximize the profit?
- Define variables
  - Z = Profit
  - x = Number of units to produce

Ex: Production of Steel

- Profit is the difference between revenue and cost (Deterministic)

$$Z = \$20x - \$5x$$

Objective Function

- Company is limited on production due to finite amount of steel

$$4x \leq 100$$

Constraint

- The objective function with the constraint is our OR model

Ex: Production of Steel

- Mathematical expression of OR model

$$\begin{aligned} &Maximize \quad Z = \$20x - \$5x \\ &subject\ to \quad 4x \leq 100 \end{aligned}$$

- We say that 20, 5, and 4 are model parameters
- Remember we are trying to find the "best" value for *x*
- Graphically, objective function is a straight line that always increases
- Why not just make 100 units? Constraint $4x \leq 100 \rightarrow x \leq 25$
- A: Set *x=25 (Large as Possible)* to maximize profit

$$Z = \$20 \times 25 - \$5 \times 25 = \$375$$

Break-Even Analysis

- Q: How many items need to be sold to make $0 in profit (break even)?

- Typical production involves fixed cost and variable cost
  - Fixed cost (c_f) independent on number of units (x)
  - Variable cost (c_v) dependent of x

- Formula for total cost C

$$C = c_f + c_v x$$

- Updated representation for profit with selling price per unit p

$$Z = px - \left(c_f + c_v x\right) = px - c_f - c_v x$$

Break-Even Analysis

- Break-even point (x^*)

$$0 = px^* - c_f - c_v x^*$$
$$0 = (p - c_v)x^* - c_f$$
$$c_f = (p - c_v)x^* \longrightarrow x^* = \frac{c_f}{p - c_v}$$

- A: If we sell x^* items, we will make \$0 in profit

Ex: Production of Jeans

- Jeanealogy creates dope jeans
- They pay $10,000 to run their factory
- Each pair of jeans costs $8 to make and sold at $23
- Q: What is the break-even point in their production?
- Key Variables

$$x = Number\ of\ Jeans\ Produced \qquad c_v = \$8$$

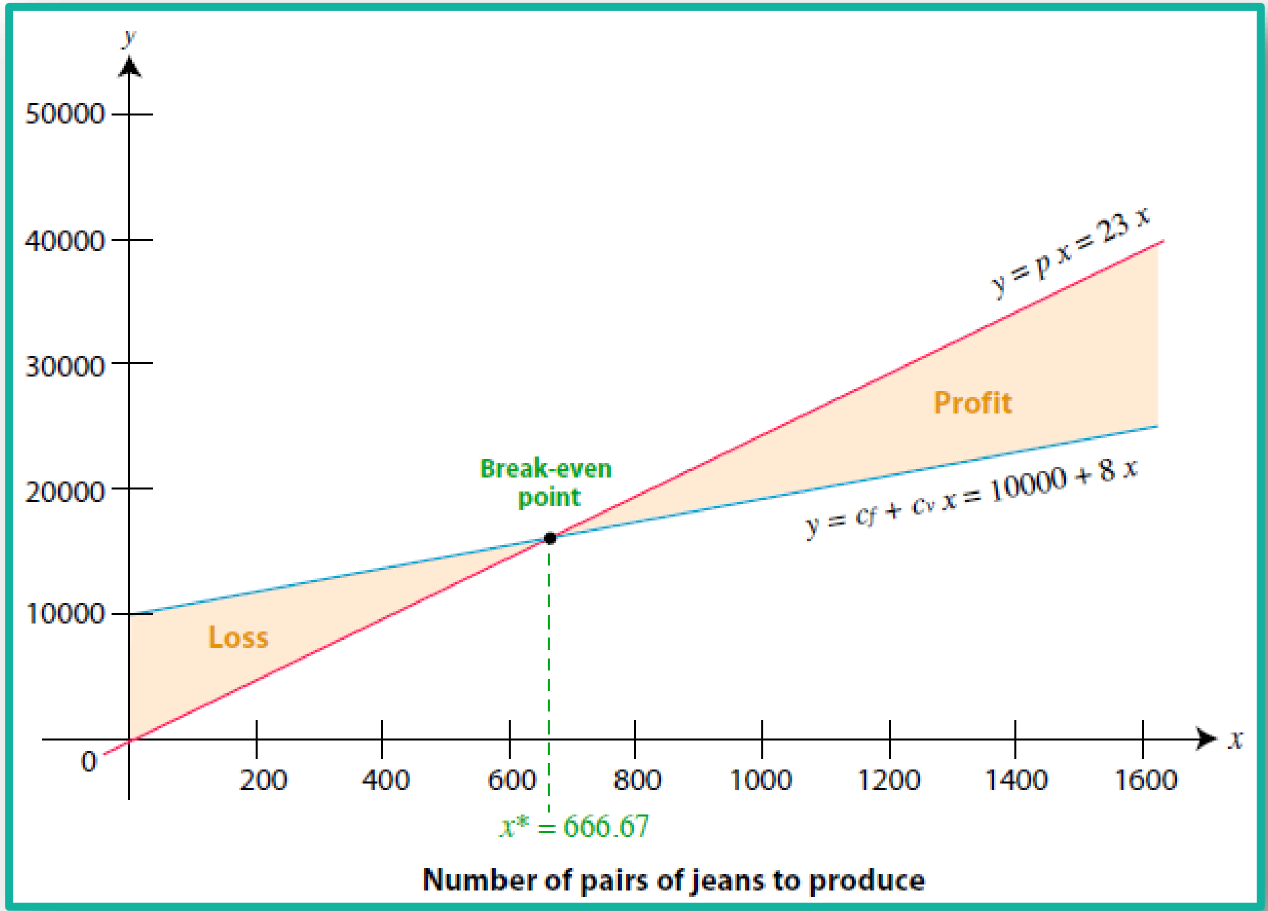
$$c_f = \$10{,}000 \qquad p = \$23$$

- A: Break-even point

$$x^* = \frac{c_f}{p - c_v} = \frac{10{,}000}{23 - 8} = 666.67 \approx 667$$

Ex: Production of Jeans

- Finding the break-even point graphically



Ex: Production of Jeans

- Sensitivity analysis is seeing how x^* is influenced by parameters c_f, c_v, and p
- Suppose Jeanology improves quality of jeans which now cost \$12 per jeans to produce but can be sold at the insane price of \$30
- Q: How does the break-even point change?
- A:

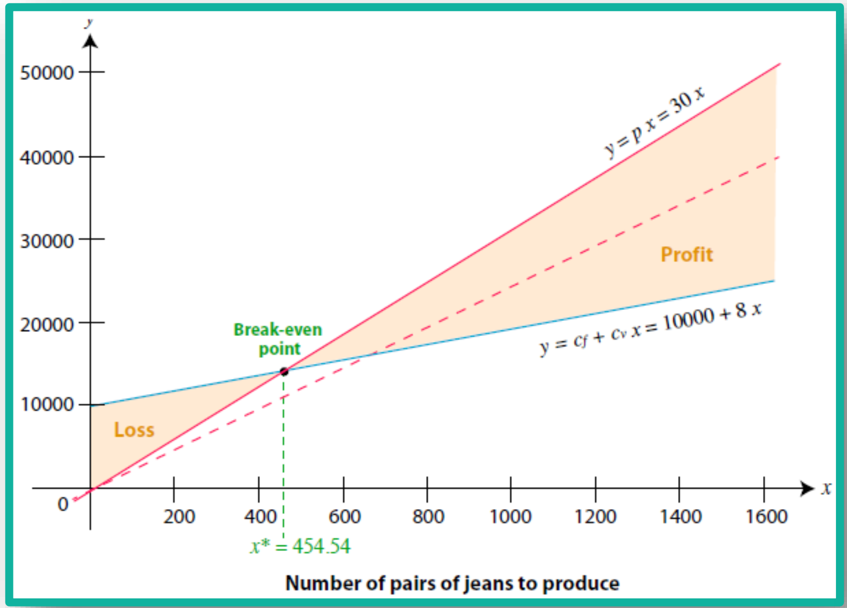
$$x^* = \frac{c_f}{p - c_v} = \frac{10{,}000}{30 - 8} = 454.54$$

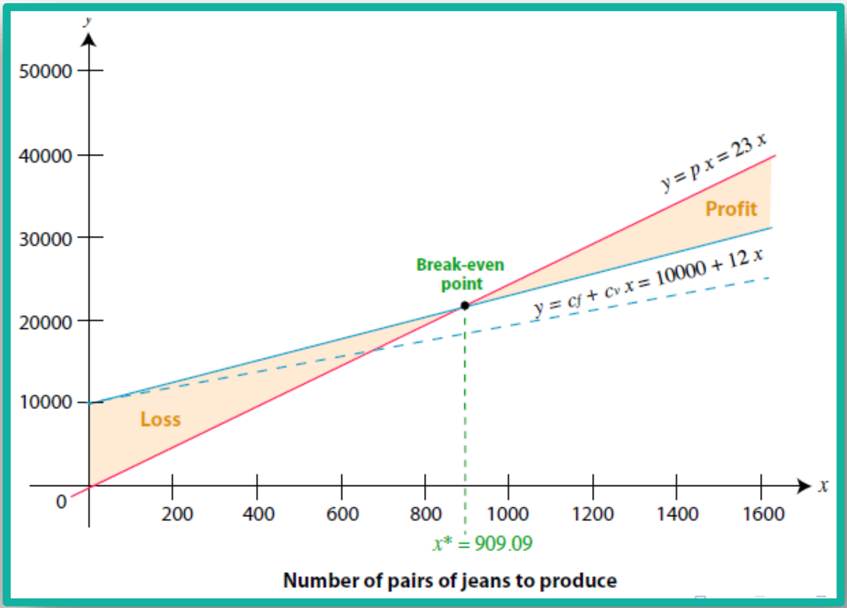
$$x^* = \frac{c_f}{p - c_v} = \frac{10{,}000}{23 - 12} = 909.09$$

$$x^* = \frac{c_f}{p - c_v} = \frac{10{,}000}{30 - 12} = 555.56$$

Ex: Production of Jeans

- Graphical sensitivity analysis





Sensitivity Analysis Motivation

- Business environment is dynamic, and parameters will change over time.
- We need to see where small changes have big effects
- Parameters are often estimated, and the break-even solution is inexact
- Businesses want to know how reliable the solution is and what is the impact of deviations from expectation

The End

Dale