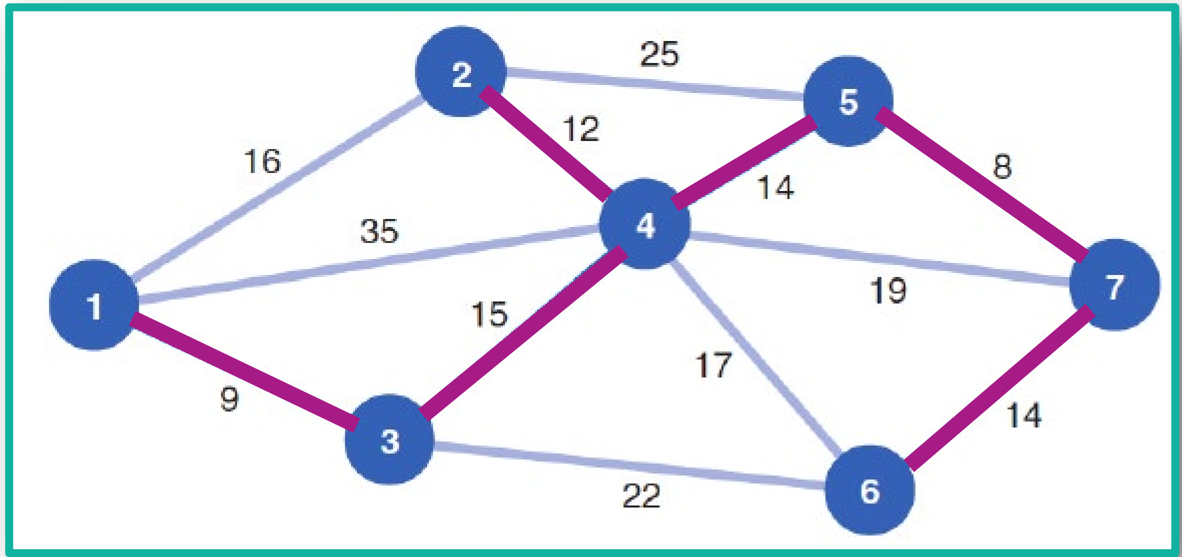
Lecture 14

Produced by Dr. Worldwide

Welcome to the 305

Ex: Cable Company

- Recall the solution the minimal spanning tree problem for the cable example



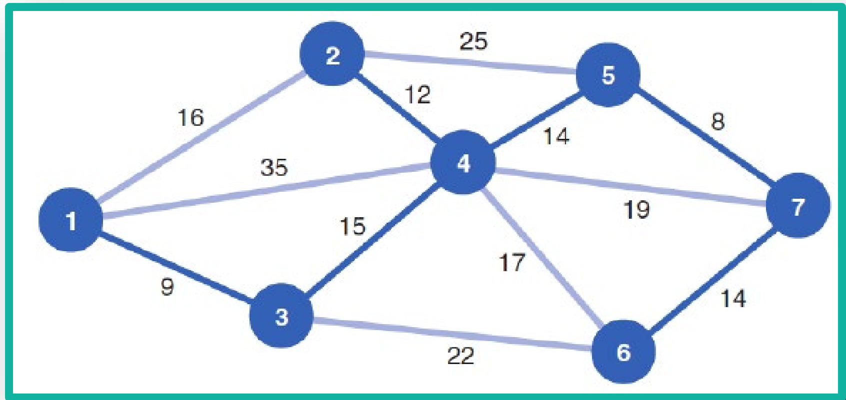
- Length of the minimal spanning tree is the sum of lengths of chosen edges

$$9 + 15 + 12 + 14 + 8 + 14 = 72$$

Ex: Cable Company

- Decision variables
  - $x_{ij} = indicator\ if\ edge\ (i,j)\ is\ selected$
  - $i = \{1,2,3,\cdots,6\}$
  - $j = \{2,3,\cdots,7\}$
  - $i < j$

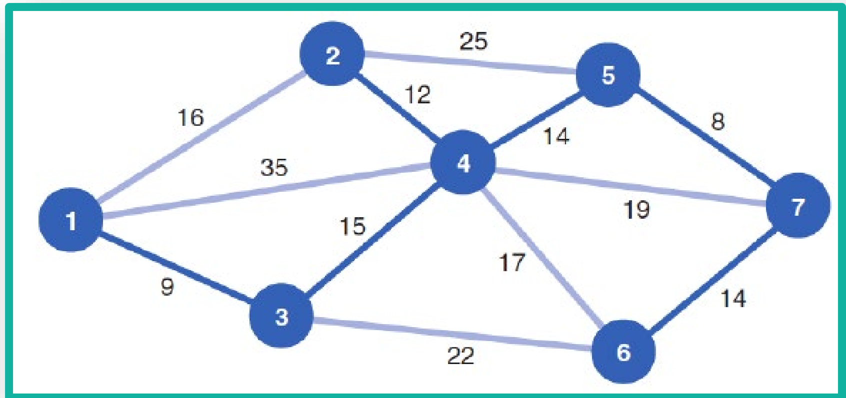
- Objective function
  - We want to minimize total distance
  - Let d_{ij} represent the distance along (i,j)
  - $Z = \sum_{i=1}^{6}\sum_{j=i+1}^{7} d_{ij}x_{ij}$



Ex: Cable Company

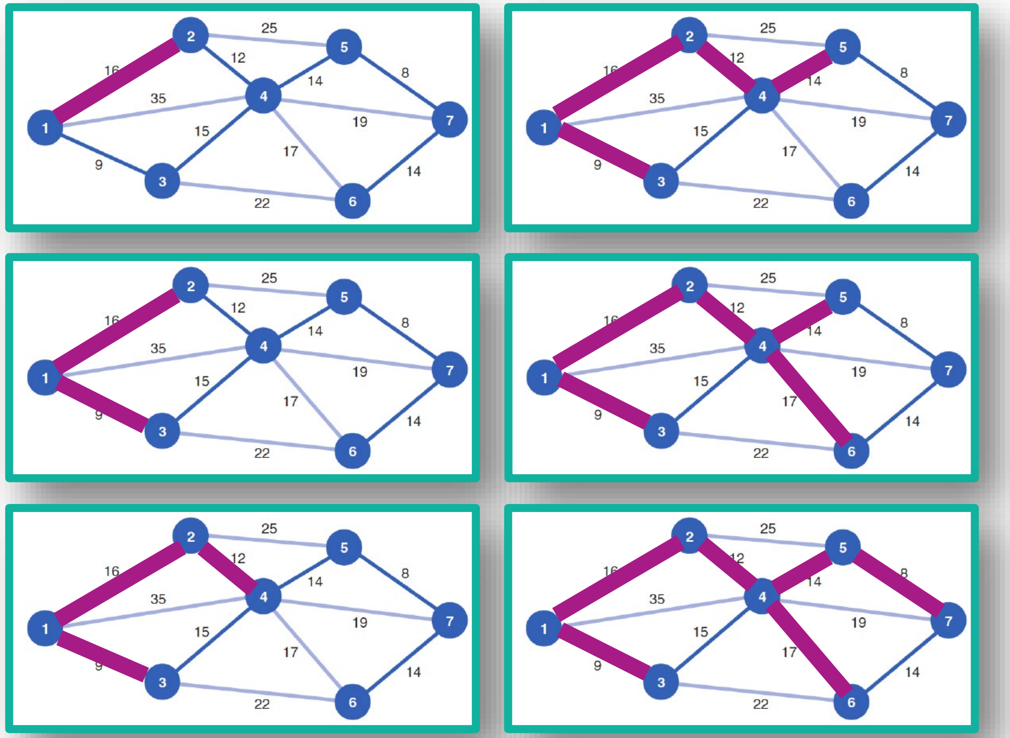
MIAMI

- Constraint to ensure each node selected at least once
  - $x_{12} + x_{14} + x_{13} \geq 1$ (Node 1)
  - $x_{12} + x_{24} + x_{25} \geq 1$ (Node 2)
  - $x_{13} + x_{34} + x_{36} \geq 1$ (Node 3)
  - $x_{14} + x_{24} + x_{34} + x_{45} + x_{46} + x_{47} \geq 1$ (Node 4)
  - $x_{25} + x_{45} + x_{57} \geq 1$ (Node 5)
  - $x_{36} + x_{46} + x_{67} \geq 1$ (Node 6)
  - $x_{47} + x_{57} + x_{67} \geq 1$ (Node 7)



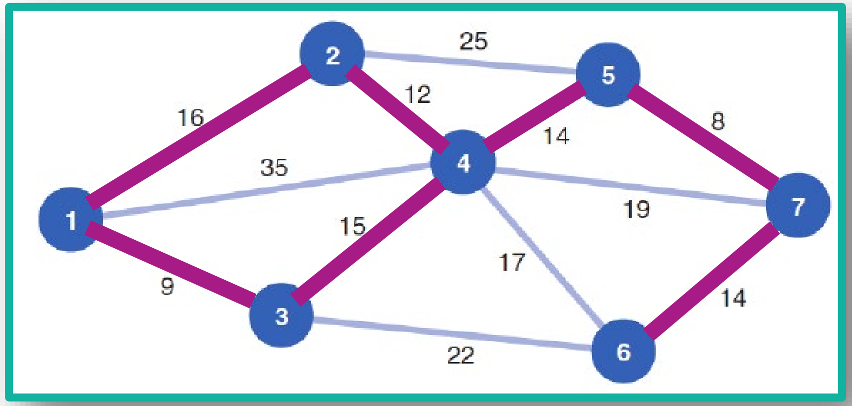
Ex: Cable Company

- Observe the following pattern of spanning trees (possibly minimal)

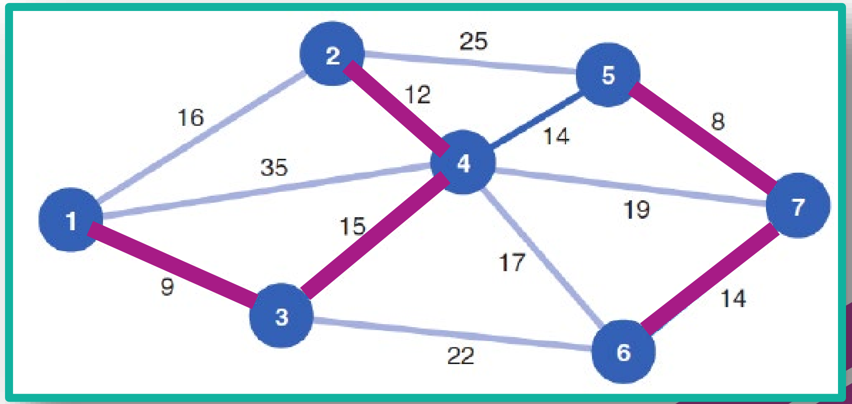


Ex: Cable Company

- Problematic cases that violate pattern
  - Case 1: Looping spanning tree



  - Case 2: Disconnected spanning trees



Ex: Cable Company

- Constraint to ensure spanning tree identified
  - Cable company needs to connect a network of 7 nodes
  - Exactly 7-1=6 branches need to be selected for a spanning tree
  - $\sum_{i=1}^{6} \sum_{j=i+1}^{7} x_{ij}=6$

- Computer solution is difficult without Excel QM

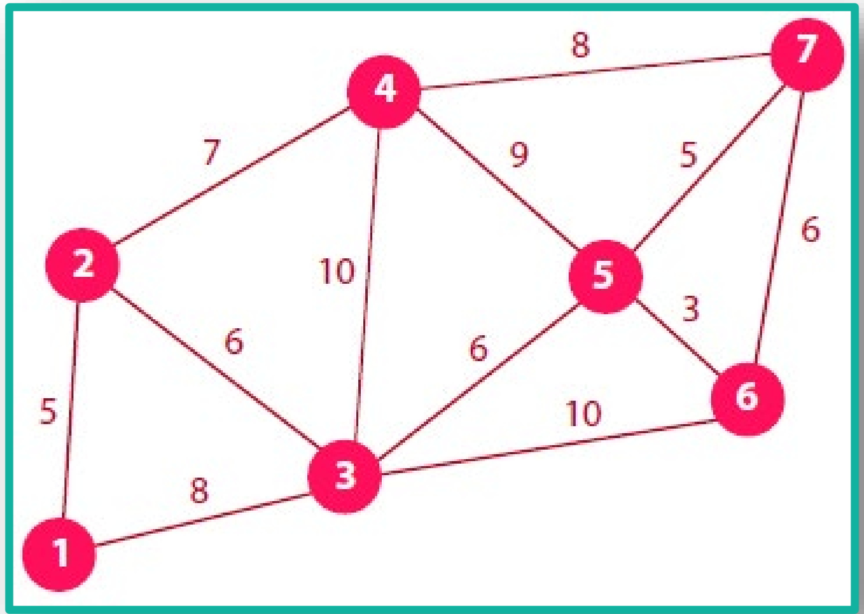
- Difficult to ensure that every subset of *k* vertices has at most k-1 edges

Overview of Network Models

- Three types of problems related to graphs with weighted edges
  - Shortest route
  - Maximal flow
  - Minimal spanning tree
- All three problems had custom algorithms and linear programming formulations
- Shortest route and minimal spanning tree are similar but give different solutions

General Problem

- Network of 7 nodes with undirected edges

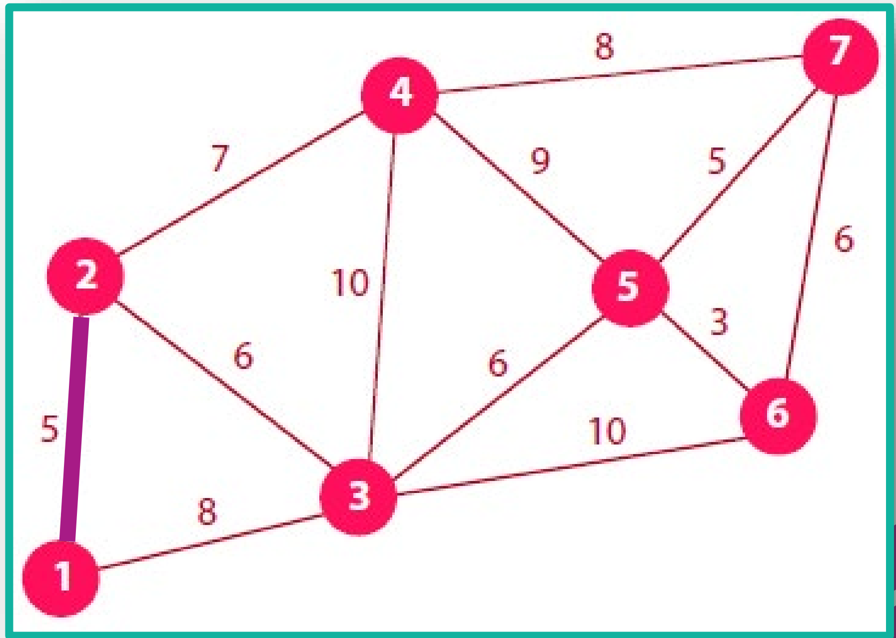


- Q: What is the shortest route from node 1 to all other nodes?
- Q: What is the minimal spanning tree of the network?

General Problem

- Shortest route algorithm
  - Start with permanent set {1}
  - Shortest adjacent node to node 1 is node 2 along edge (1,2)
  - Add node 2 to permanent set
  - Keep track of edge (1,2)

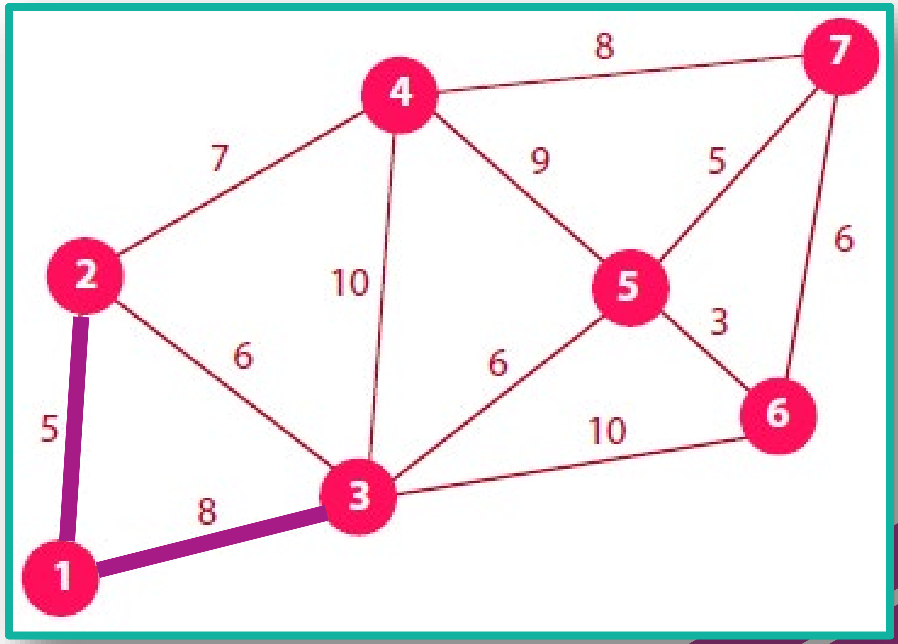
Set	Path	Distance
{1}	1-2	$\boxed{5}$
	1-3	8



General Problem

- Shortest route algorithm
  - New permanent set {1,2}
  - Shortest node to node 1, not in permanent set, is node 3
  - Add node 3 to permanent set
  - Keep track of edge (1,3)

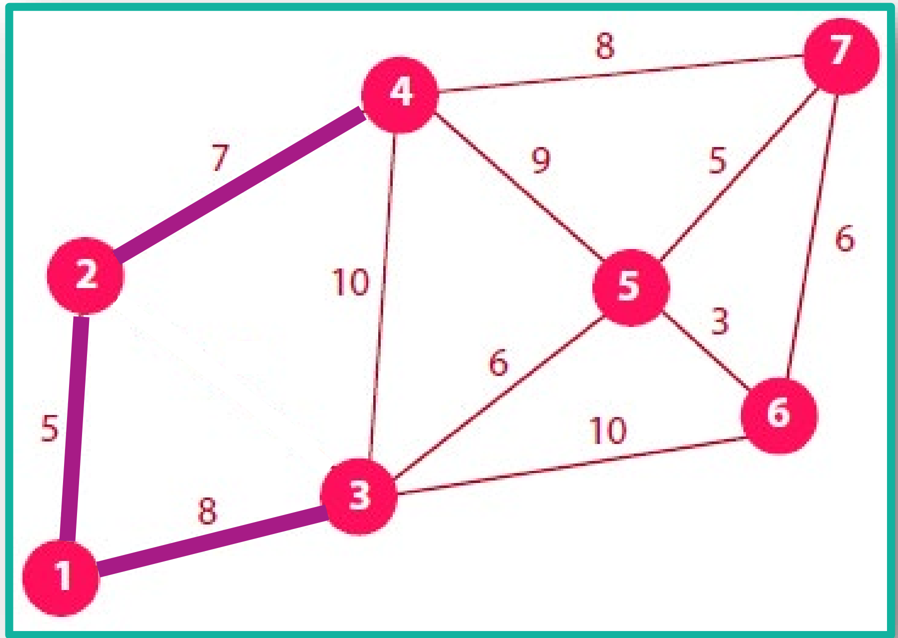
Set	Path	Distance
{1}	1-2	5
{1,2}	1-3	$\boxed{8}$
	1-2-3	11
	1-2-4	12



General Problem

- Shortest route algorithm

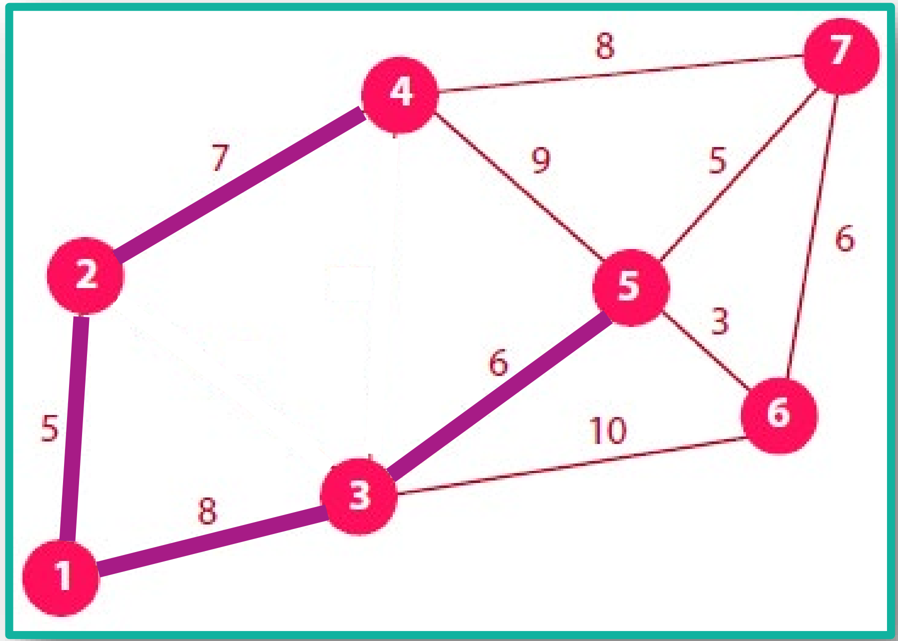
Set	Path	Distance
{1}	1-2	5
{1,2}	1-3	8
{1,2,3}	1-2-4	$\boxed{12}$
	1-3-4	18
	1-3-5	14
	1-3-6	18



General Problem

- Shortest route algorithm

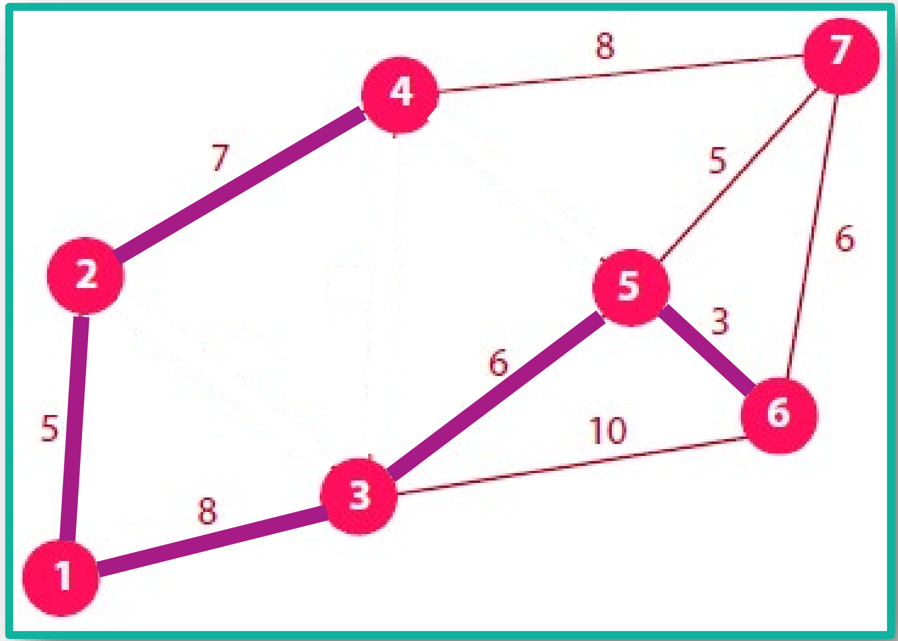
Set	Path	Distance
{1}	1-2	5
{1,2}	1-3	8
{1,2,3}	1-2-4	12
{1,2,3,4}	1-2-4-5	21
	1-3-5	$\boxed{14}$
	1-3-6	18
	1-2-4-7	20



General Problem

- Shortest route algorithm

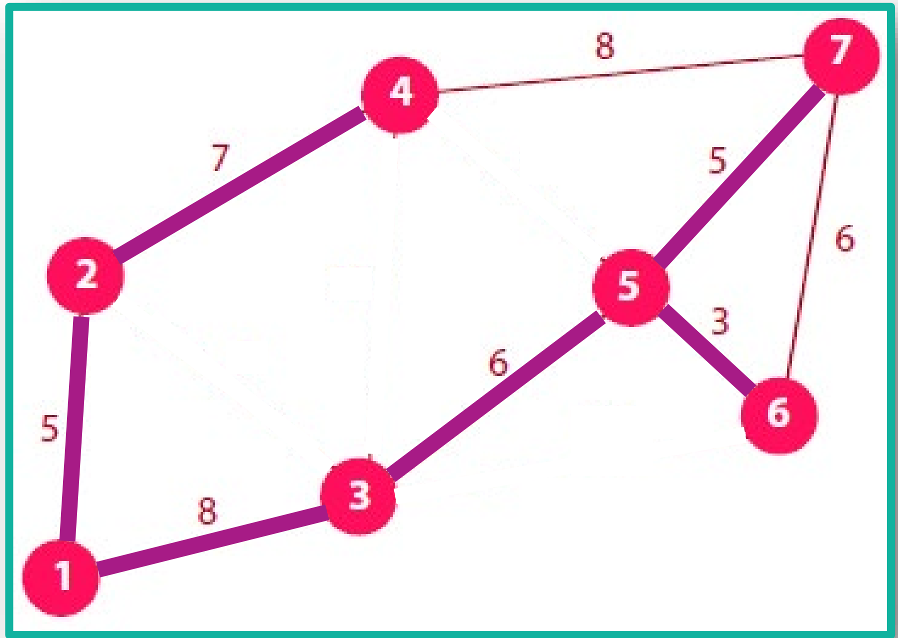
Set	Path	Distance
{1}	1-2	5
{1,2}	1-3	8
{1,2,3}	1-2-4	12
{1,2,3,4}	1-3-5	14
{1,2,3,4,5}	1-3-6	18
	1-3-5-6	17
	1-2-4-7	20
	1-3-5-7	19



General Problem

- Shortest route algorithm

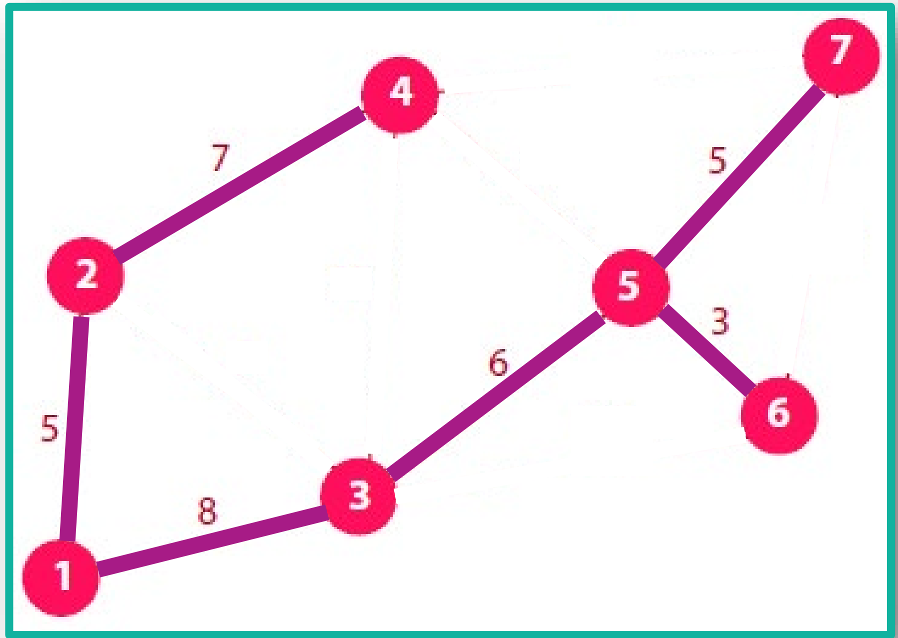
Set	Path	Distance
{1}	1-2	5
{1,2}	1-3	8
{1,2,3}	1-2-4	12
{1,2,3,4}	1-3-5	14
{1,2,3,4,5}	1-3-5-6	17
{1,2,3,4,5,6}	1-2-4-7	20
	1-3-5-6-7	23
	1-3-5-7	$\boxed{19}$



General Problem

- Shortest route algorithm

Closest	Path	Distance
2	1-2	5
3	1-3	8
4	1-2-4	12
5	1-3-5	14
6	1-3-5-6	17
7	1-3-5-7	19



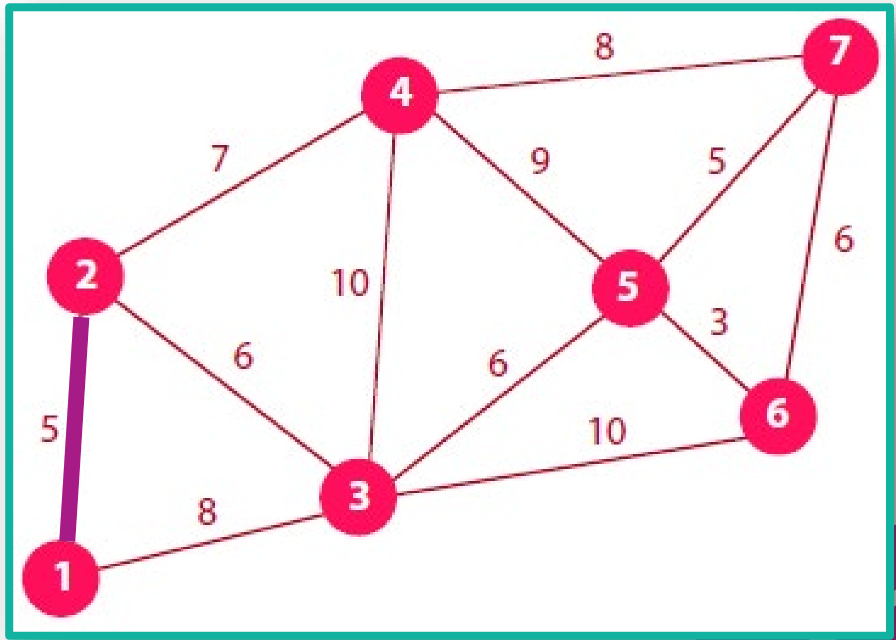
- The sum of all the edges from the final spanning tree

$5 + 7 + 8 + 6 + 3 + 5 = 34$

General Problem

- Minimal spanning tree algorithm
  - Start with node 1 in spanning set {1}
  - Shortest adjacent node to any node in spanning set is node 2
  - Add node 2 to spanning set
  - Keep track of edge (1,2)

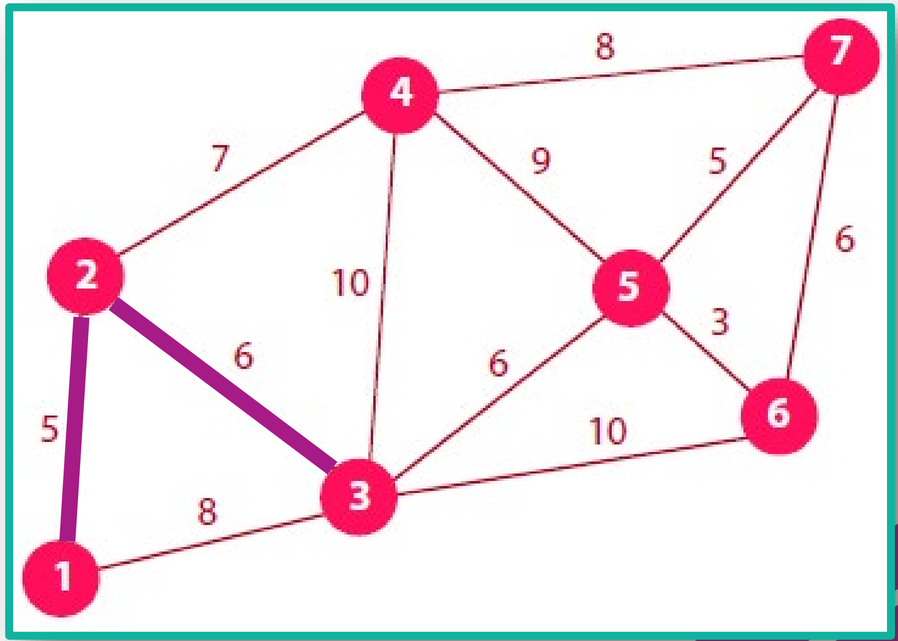
Set	Path	Distance
{1}	1-2	$\boxed{5}$
	1-3	8



General Problem

- Minimal spanning tree algorithm
  - Consider spanning set {1,2}
  - Shortest adjacent node to any node in spanning set is node 3
  - Add node 3 to spanning set
  - Keep track of edge (2,3)

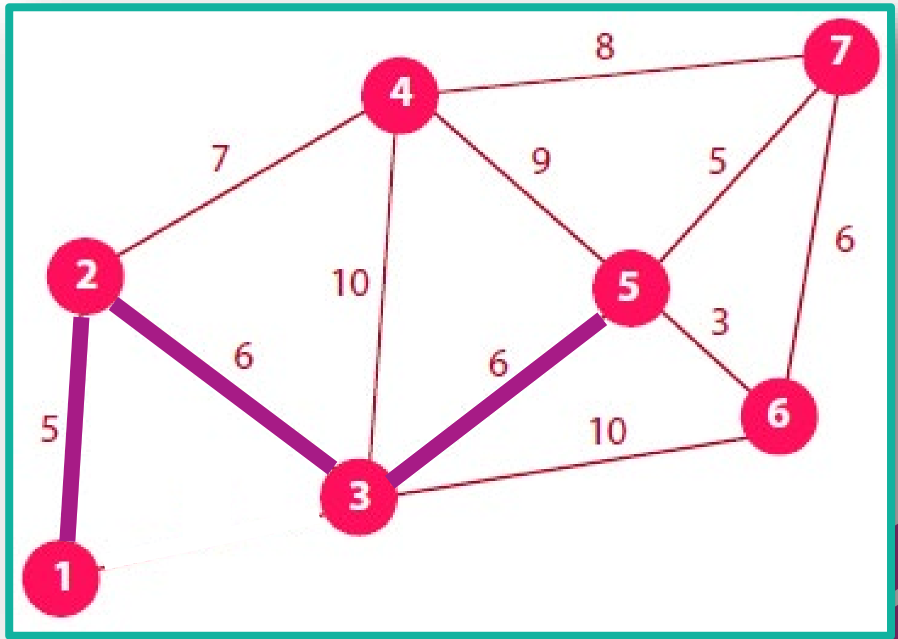
Set	Path	Distance
{1}	1-2	5
{1,2}	1-3	8
	2-3	6
	2-4	7



General Problem

- Minimal spanning tree algorithm
  - Consider spanning set {1,2,3}
  - Shortest adjacent node to any node in spanning set is node 5
  - Add node 5 to spanning set
  - Keep track of edge (3-5)

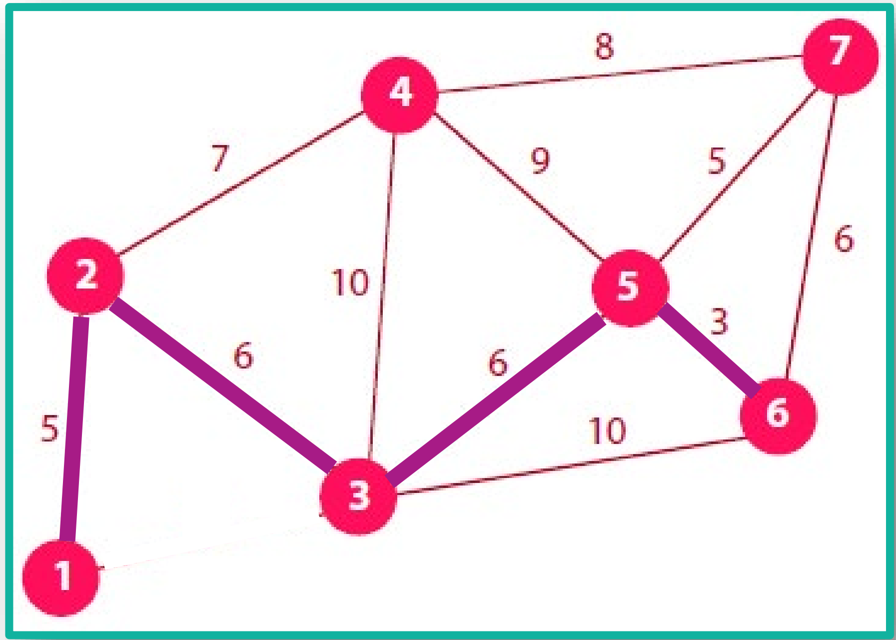
Set	Path	Distance
{1}	1-2	5
{1,2}	2-3	6
{1,2,3}	2-4	7
	3-4	10
	3-5	$\boxed{6}$
	3-6	10



General Problem

- Minimal spanning tree algorithm

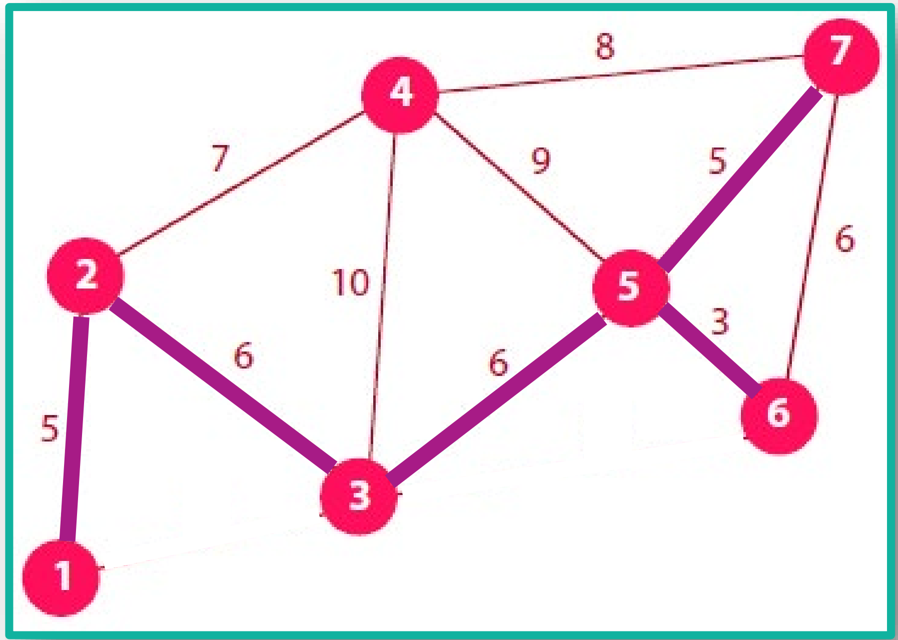
Set	Path	Distance
{1}	1-2	5
{1,2}	2-3	6
{1,2,3}	3-5	6
{1,2,3,5}	2-4	7
	3-4	10
	5-4	9
	3-6	10
	5-6	$\boxed{3}$



General Problem

- Minimal spanning tree algorithm

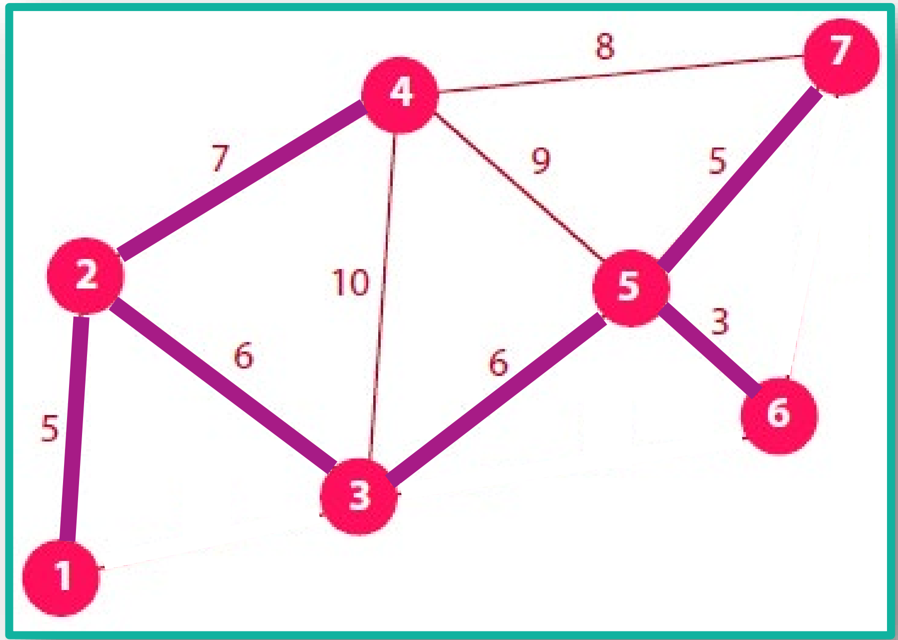
Set	Path	Distance
{1}	1-2	5
{1,2}	2-3	6
{1,2,3}	3-5	6
{1,2,3,5}	5-6	3
{1,2,3,5,6}	2-4	7
	3-4	10
	5-4	9
	5-7	$\boxed{5}$
	6-7	6



General Problem

- Minimal spanning tree algorithm

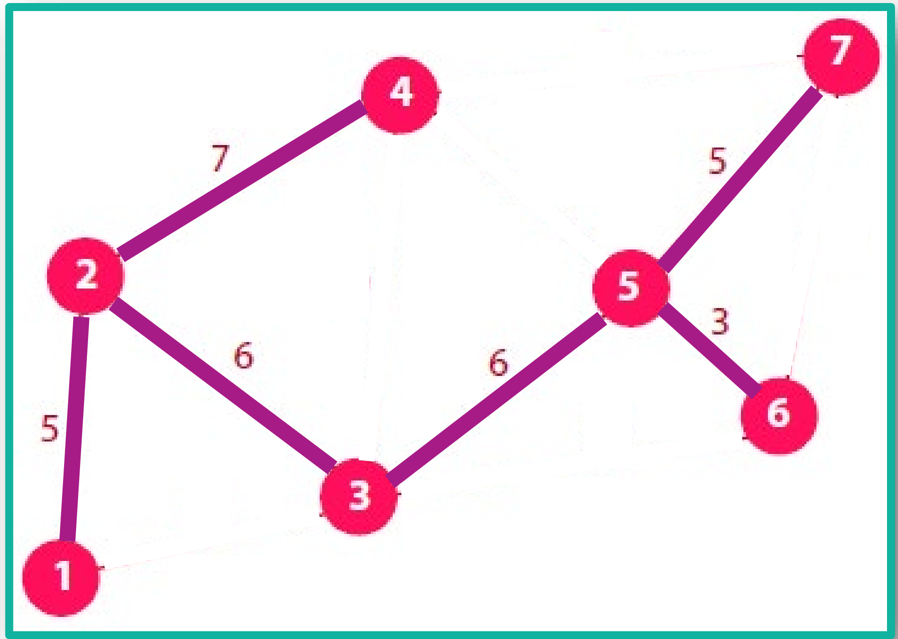
Set	Path	Distance
{1}	1-2	5
{1,2}	2-3	6
{1,2,3}	3-5	6
{1,2,3,5}	5-6	3
{1,2,3,5,6}	5-7	5
{1,2,3,5,6,7}	2-4	$\boxed{7}$
	3-4	10
	5-4	9
	7-4	8



General Problem

- Minimal spanning tree algorithm

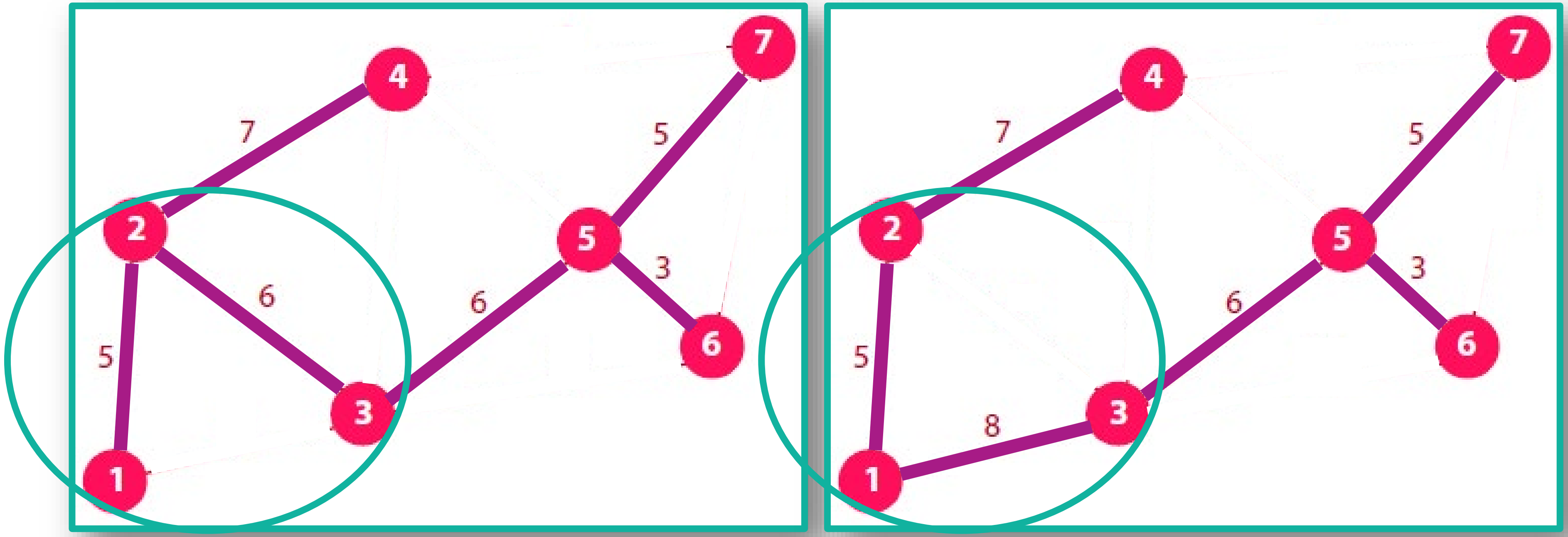
Added	Path	Distance
2	1-2	5
3	2-3	6
5	3-5	6
6	5-6	3
7	5-7	5
4	2-4	7



- The sum of all the edges from the minimal spanning tree
  $5 + 7 + 6 + 6 + 3 + 5 = 32$

General Problem

- Solution comparison



- Different solutions because algorithms target different goals

The End

Dale