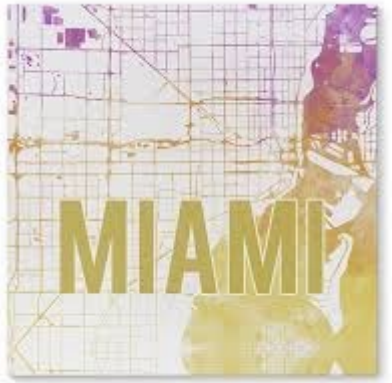
Lecture 16

Produced by Dr. Worldwide

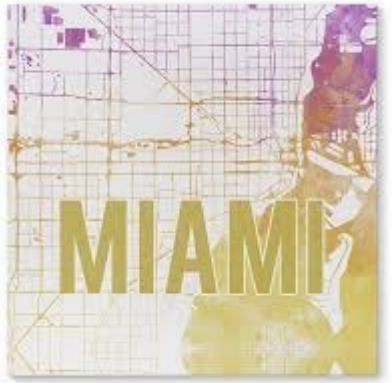
Welcome to the 305

Excel for Goal Programming



- Builds off linear programming using Excel Solver
- Solve the linear program multiple times with different objective functions
- Go in order of priority
- After finding the optimal solution, we add the optimal value attained in the first objective function as a new constraint and move on to the next objective function
- Possible that while solving for a given priority, we simultaneously optimize other lower ranked priorities

Ex: Beaver Creek Pottery



- Download GoalProgramming.xlsx from link Sheet 1 on course website

- See tab Priority 1 for minimization of d_1^-
  - Optimal solution

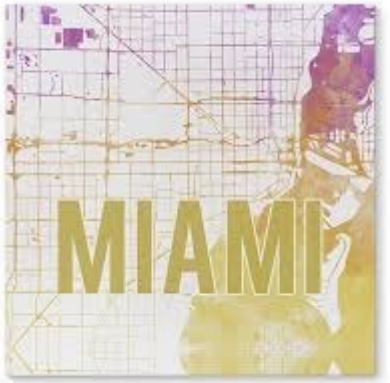
    $x = 15 \quad y = 20 \quad d_1^+ = 15 \quad d_4^+ = 5 \quad d_5^- = 15$
    $d_1^-, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_6^- = 0$

  - It is optimal to set $d_1^- = 0$
  - In our system of linear constraints, we have employees working at least 40 hr

- Move on to P2 for minimization of d_2^-
  - Notice from last solution $d_2^- = 0$
  - Optimal solution from P1 minimizes objective function from P2

- Unnecessary to consider P3 since $d_3^+ = 0$ under optimal solution of P1

Ex: Beaver Creek Pottery



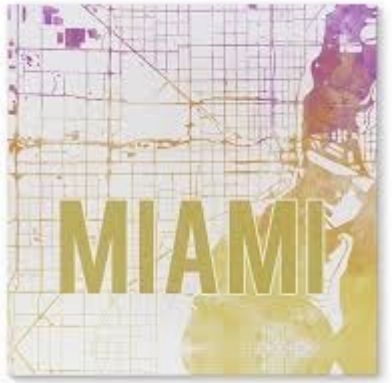
- See tab Priority 4 for minimization of d_4^+
  - To ensure none of the optimal values achieved thus far change when we attempt to minimize d_4^+, we add the values attained as constraints
  - We add one constraint for each goal we have already attained

Minimize d_4^+

Subject to

$$x + 2y + d_1^- - d_1^+ = 40 \quad \text{(Labor)}$$
$$40x + 50y + d_2^- - d_2^+ = 1600 \quad \text{(Profit)}$$
$$4x + 3y + d_3^- - d_3^+ = 120 \quad \text{(Clay)}$$
$$d_1^+ + d_4^- - d_4^+ = 10 \quad \text{(Overtime)}$$
$$x + d_5^- = 30 \quad \text{(Bowls)}$$
$$y + d_6^- = 20 \quad \text{(Mugs)}$$
$$d_1^-, d_2^-, d_3^+ = 0$$
$$x, y, d_1^+, d_2^+, d_3^-, d_4^-, d_4^+, d_5^-, d_6^- \geq 0$$

Ex: Beaver Creek Pottery

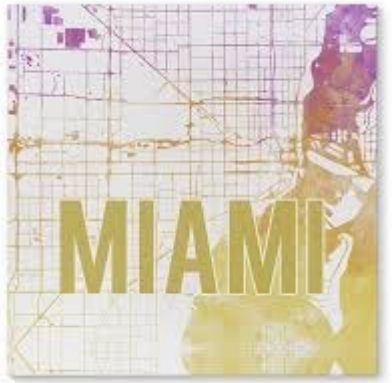


- See tab Priority 4 for minimization of d_4^+
  - Optimal solution

    $x = 15 \quad y = 20 \quad d_1^+ = 15 \quad d_4^+ = 5 \quad d_5^- = 15$
    $d_1^-, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_6^- = 0$

  - Solution did not change and $d_4^+ = 5$ in both cases
  - Not possible to reduce the value of d_4^+ without violating the optimal solutions for the three goals that have higher priority
  - This indicates that the overtime must be exceed by 5 hours to fulfill other constraints from higher priority goals

Ex: Beaver Creek Pottery



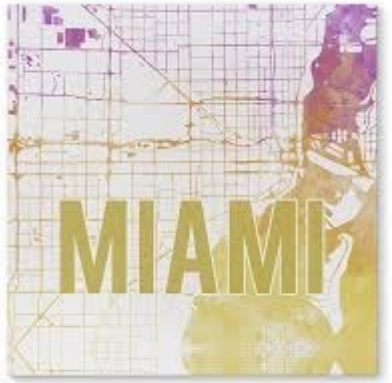
- See tab Priority 5 for minimization of $4d_5^- + 5d_6^-$
  - Add result from previous priority rank as a constraint

Minimize $4d_5^- + 5d_6^-$

Subject to

$$x + 2y + d_1^- - d_1^+ = 40 \quad \text{(Labor)}$$
$$40x + 50y + d_2^- - d_2^+ = 1600 \quad \text{(Profit)}$$
$$4x + 3y + d_3^- - d_3^+ = 120 \quad \text{(Clay)}$$
$$d_1^+ + d_4^- - d_4^+ = 10 \quad \text{(Overtime)}$$
$$x + d_5^- = 30 \quad \text{(Bowls)}$$
$$y + d_6^- = 20 \quad \text{(Mugs)}$$
$$d_1^-, d_2^-, d_3^+ = 0 \quad \text{(New Constraints)}$$
$$d_4^+ = 5 \quad \text{(New Constraints)}$$
$$x, y, d_1^+, d_2^+, d_3^-, d_4^-, d_5^-, d_6^- \geq 0$$

Ex: Beaver Creek Pottery



- See tab Priority 5 for minimization of $4d_5^- + 5d_6^-$
  - Optimal solution

    $x = 15 \quad y = 20 \quad d_1^+ = 15 \quad d_4^+ = 5 \quad d_5^- = 15$
    $d_1^-, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_6^- = 0$

  - Solution still did not change
  - Optimal solution stays optimal
- Final Solution
  - Produce 15 bowls and 20 mugs
  - Hours of work: $15 + 2(20) = 55$ (Over by 15 hours)
  - Profit: $40(15) + 50(20) = 1600$
  - Pounds of clay: $4(15) + 3(20) = 120$
  - Overtime beyond 10 hours: $d_4^+ = 5$
  - Slack for bowls below 30: $d_5^- = 15$
  - Slack for mugs below 20: $d_4^+ = 0$

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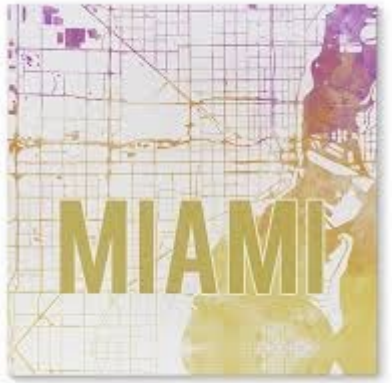

- Full modified goal programming model

Minimize $P_1 d_1^-, P_2 d_2^-, P_3 d_3^+, P_4 d_4^+, P_5(4d_5^- + 5d_6^-)$

Subject to

$$x + 2y + d_1^- - d_1^+ = 40 \quad \text{(Labor)}$$
$$40x + 50y + d_2^- - d_2^+ = 1600 \quad \text{(Profit)}$$
$$4x + 3y + d_3^- - d_3^+ = 120 \quad \text{(Clay)}$$
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$$x + d_5^- = 30 \quad \text{(Bowls)}$$
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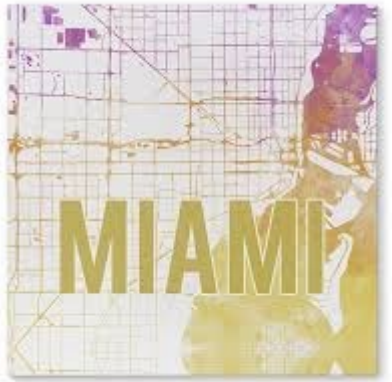
$$x, y, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+, d_5^-, d_6^- \geq 0$$

Ex: The Expanse



- In the Expanse, there are three main factions:
  - Earthers from Earth
  - Martians from Mars
  - Belters from the Asteroid Belt

- All three factions have fought for years and the well-respected earther, Jim Holden, who works in the belt, has recommended Earth and Mars to balance representation of their people at 4 asteroid developments in the belt

- Current distribution at the 4 space stations
  - Ceres: 500 Earthers & 500 Martians
  - Vesta: 300 Earthers & 1,000 Martians
  - Pallas: 400 Earthers & 1,050 Martians
  - Hygiea: 800 Earthers & 450 Martians

- Overall: 5,000 Earthers and Martians: 40% Earthers & 60% Martians

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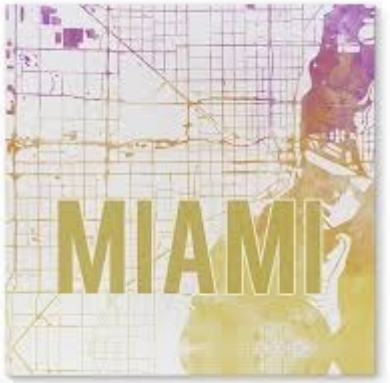

- Currently, Ceres is the only asteroid that has even balance between people from Earth and Mars

- Six prominent individuals from Mars and Earth, who are leaders at the various asteroids, come together to determine the best way to relocate their people
  - John Connor (JC) – Represents Pallas
  - Fred Harvey (FH) – Represents Ceres
  - Betty Philips (BP) – Represents Vesta
  - Mickey Gibbony (MG) – Represents Hygiea
  - Cassandra Watkins (CW) – Represents Ceres
  - Bob Wilson (BW) – Represents Pallas

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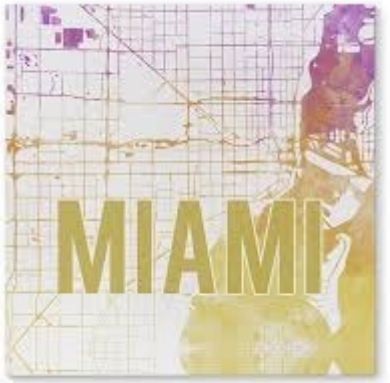
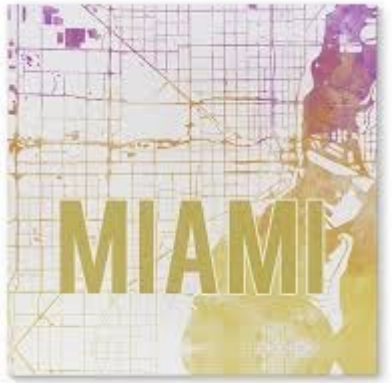


- Act I
  - "Rather than starting off by trying to move people from one asteroid to another, why don't we try to establish what we want to accomplish?" – JC
  - "Good idea, Lil John" – FH
  - "Sixty percent of all residents are Martians and 40% are Earthers, so that's what we need our asteroids to be, 60% and 40%." – JC
  - "That's okay for you to say, Lil John, because your asteriod (Pallas) is already close to those proportions. My asteroid (Vesta) is a long way from that ratio, and we would have to move a lot of our residents." – BP
  - "I'm not saying it, Betty; Jim Holden has been saying it for 6 months." – JC
  - "John's right, Betty, and we're not moving people yet; we're just putting down our objectives. I think that must be our highest-priority." – FH
  - *They all nod in agreement like a bunch of bobbleheads*

- Intermission I

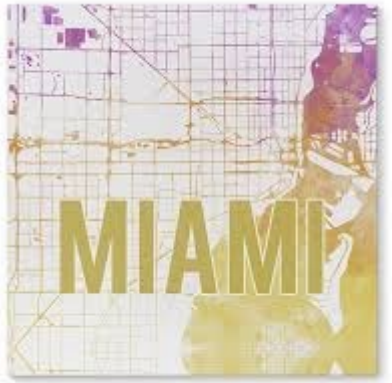
Ex: The Expanse



- Act II
  - "Since we're going to have to move people to achieve this ratio at each asteroid, I think we ought to try to minimize the amount of traveling."– MG
  - *Fred Harvey shared a table showing the distance (in millions of miles) a person in one asteroid would have to travel to get to each of the other asteroids.*

Asteroid	Vesta	Hygiea	Pallas	Ceres
Vesta	-	30	12	20
Hygiea	30	-	18	26
Pallas	12	18	-	24
Ceres	20	26	24	-

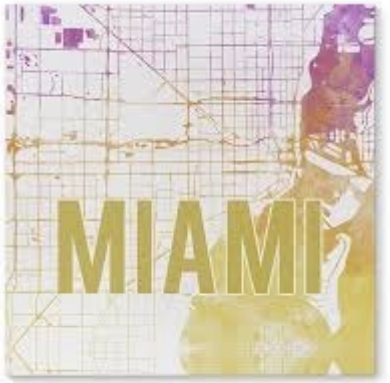
Ex: The Expanse



- Act II
  - "Why not set reasonable objectives for total miles, for the peoples' sake and for budgeting reasons? I would suggest about 30,000 million miles of total travel at max. If we get much higher than that we're not going to have the money to pay for it, and it means we'll be moving people all over the place." – CW
  - *They all nod in agreement like a bunch of bobbleheads*
  - "Okay, that'll be our number two goal." – FH

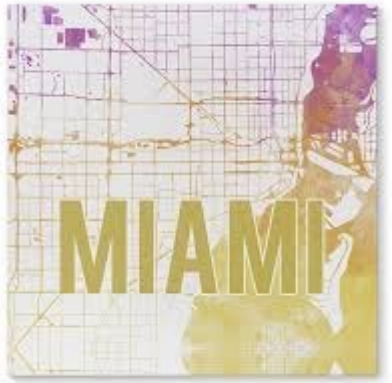
- Intermission II

Ex: The Expanse



- Act III
  - "I'll tell you another thing I don't want to see happen, and that's more overcrowding at Vesta. We have 100 people more than capacity now." – BP
  - "You think you have problems! In Pallas, we have 1,450 people and a capacity of 1,000. I think no overcrowding is a great idea!" – BW
  - "I agree. We're 250 over our capacity at Hygiea" – MG
  - "That's a nice idea, and I realize that we have 200 people less than our capacity at Ceres. However, let's face it, across the asteroids we have capacity for 4,400, not 5,000, people, so there's going to be some overcrowding. I think our objective should be that all 4 asteroids should share in the overcrowding proportionally." – CW
  - "That sounds reasonable to me. How about the rest of you? Okay to say our number three goal is to be as close to capacity at each asteroid as possible but share proportionally in the overcrowding" - FH
  - *They voice their approval by joining hands and singing*

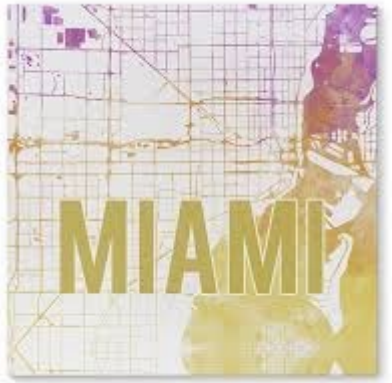
Ex: The Expanse



- Act III
  - "Well, I think we have identified the things we want to accomplish in our plan. Now, if we could just use some magic trick to find the best way to move these people to achieve these goals." – JC
  - *The others nodded and frowned because they don't know math.*

- The End and Credits
  - John Connor — *Mario Giacomazzo*
  - Fred Harvey — *Mario Giacomazzo*
  - Betty Philips — *Mario Giacomazzo*
  - Mickey Gibbony — *Mario Giacomazzo*
  - Cassandra Watkins — *Mario Giacomazzo*
  - Bob Wilson — *Mario Giacomazzo*
  - Narration — *Mario Giacomazzo*

Ex: The Expanse

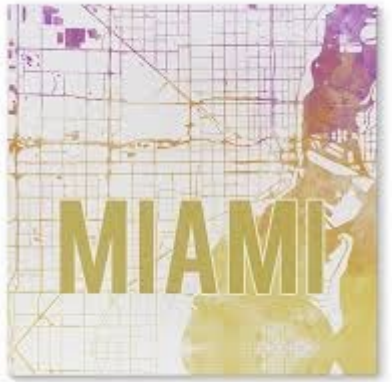


- Goals listed in order based on priority
  - Achieve a 60%/40% ratio of Martians to Earthers at each of the asteroids
  - Minimize the amount of traveling that people will have to do, ideally no more than 30,000 million miles
  - Keep all asteroids close to capacity and minimize overcrowding proportionally allocating the excess among the asteroids

- Q: How can we formulate and solve a goal programming model to help these representatives with their dilemma?

- Decision variables
  - x_{ij} = Number of martians from asteroid i assigned to asteroid j
  - y_{ij} = Number of earthers from asteroid i assigned to asteroid j
  - $i, j \in \{V, H, P, C\}$

Ex: The Expanse

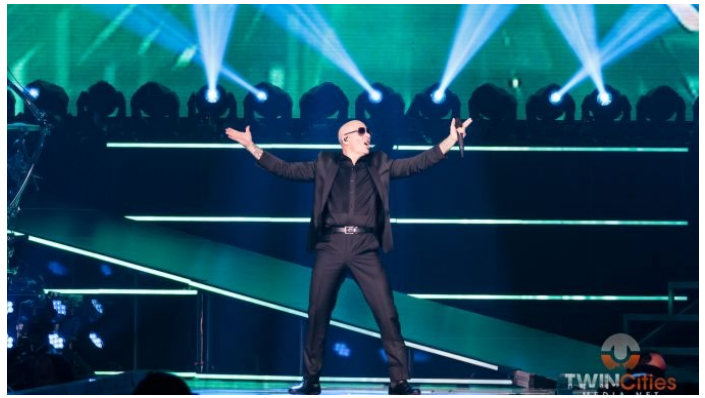


- Helpful tables of key information
  - Current amounts of Martians and Earthers with capacity

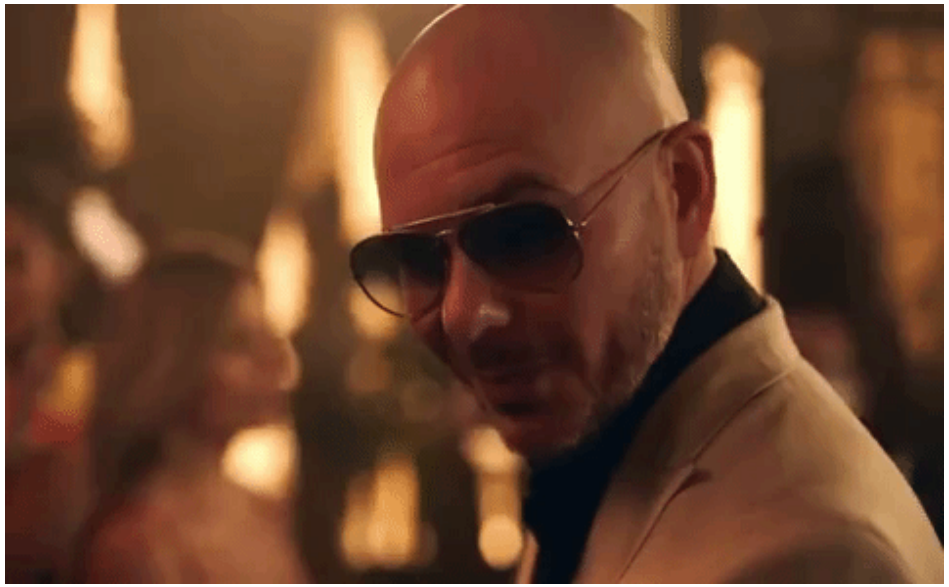
Asteroid	# of Martians	# of Earthers	*Capacity*
Vesta	1000	300	1200
Hygiea	450	800	1000
Pallas	1050	400	1000
Ceres	500	500	1200

  - Distances (millions of miles) each person must travel between asteroids

Asteroid	Vesta	Hygiea	Pallas	Ceres
Vesta	-	30	12	20
Hygiea	30	-	18	26
Pallas	12	18	-	24
Ceres	20	26	24	-



The End



Dale