Lecture 17

Produced by Dr. Worldwide

Welcome to the 305

Ex: The Expanse

- Goals listed in order based on priority
  - Achieve a 60%/40% ratio of Martians to Earthers at each of the asteroids
  - Minimize the amount of traveling that people will have to do, ideally no more than 30,000 million miles
  - Keep all asteroids close to capacity and minimize overcrowding proportionally allocating the excess among the asteroids

- Q: How can we formulate and solve a goal programming model to help these representatives with their dilemma?

- Decision variables
  - x_{ij} = Number of martians from asteroid i assigned to asteroid j
  - y_{ij} = Number of earthers from asteroid i assigned to asteroid j
  - $i, j \in \{V, H, P, C\}$

Ex: The Expanse

- Goal 1: Achieve fair representation in all 4 asteroids
  - Consider perfect balance for Vesta

$$\frac{Percent\ Martian}{Percent\ Earther} = \frac{0.6}{0.4}$$

$$\frac{\frac{Total\ Martian}{Total\ Earther}}{\frac{Total\ Martian}{Total\ Earther}} = \frac{0.6}{0.4}$$

$$\frac{Total\ Martian}{Total\ Earther} = \frac{0.6}{0.4}$$

$$0.4(Total\ Martian) - 0.6(Total\ Earther) = 0$$
$$0.4(x_{VV} + x_{HV} + x_{PV} + x_{CV}) - 0.6(y_{VV} + y_{HV} + y_{PV} + y_{CV}) = 0$$

Ex: The Expanse

- Goal 1: Achieve fair representation in all 4 asteroids
  - Adding deviational variables for Vesta

$$0.4(x_{VV}+x_{HV}+x_{PV}+x_{CV})-0.6(y_{VV}+y_{HV}+y_{PV}+y_{CV})+d_1^- - d_1^+ = 0$$

  - Consider constraints for each of the asteroids

$$\begin{aligned}
0.4(x_{VV}+x_{HV}+x_{PV}+x_{CV})-0.6(y_{VV}+y_{HV}+y_{PV}+y_{CV})+d_1^- - d_1^+ &= 0\\
0.4(x_{VH}+x_{HH}+x_{PH}+x_{CH})-0.6(y_{VH}+y_{HH}+y_{PH}+y_{CH})+d_2^- - d_2^+ &= 0\\
0.4(x_{VP}+x_{HP}+x_{PP}+x_{CP})-0.6(y_{VP}+y_{HP}+y_{PP}+y_{CP})+d_3^- - d_3^+ &= 0\\
0.4(x_{VC}+x_{HC}+x_{PC}+x_{CC})-0.6(y_{VC}+y_{HC}+y_{PC}+y_{CC})+d_4^- - d_4^+ &= 0
\end{aligned}$$

  - To accomplish our goal, we want all of these deviational variables to be as small as possible
  - First priority objective

$$\text{Minimize} \qquad P_1(d_1^- + d_1^+ + d_2^- + d_2^+ + d_3^- + d_3^+ + d_4^- + d_4^+)$$

Ex: The Expanse

- Goal 2: Minimize total travel to not much more than 30,000 million miles
  - Recall the following table in millions of miles

Asteroid	Vesta	Hygiea	Pallas	Ceres
Vesta	-	30	12	20
Hygiea	30	-	18	26
Pallas	12	18	-	24
Ceres	20	26	24	-

  - Formulation for constraint based on total miles

$$30(x_{VH} + y_{VH} + x_{HV} + y_{HV}) + 12(x_{VP} + y_{VP} + x_{PV} + y_{PV})$$
$$+20(x_{VC} + y_{VC} + x_{CV} + y_{CV}) + 18(x_{HV} + y_{HV} + x_{VH} + y_{VH})$$
$$+26(x_{HC} + y_{HC} + x_{CH} + y_{CH}) + 24(x_{PC} + y_{PC} + x_{CP} + y_{CP}) + d_5^- - d_5^+ = 30{,}000$$

  - Updated objective function for second priority

$$\text{Minimize} \qquad P_1(d_1^- + d_1^+ + d_2^- + d_2^+ + d_3^- + d_3^+ + d_4^- + d_4^+), P_2(d_5^+)$$

Ex: The Expanse

- Goal 3: Minimize overcrowding at each asteroid, proportionally allocating the excess among the asteroids
  - Recall the following table

Asteroid	# of Martians	# of Earthers	*Capacity*
Vesta	1000	300	1200
Hygiea	450	800	1000
Pallas	1050	400	1000
Ceres	500	500	1200

  - Recall that there are 5,000 total people for capacity of 4,400
  - The excess of 600 people needs to be split between the asteroids
  - Q: How can we handle this proportionally?

Ex: The Expanse

- Goal 3: Minimize overcrowding at each asteroid, proportionally allocating the excess among the asteroids
  - We want to manage the excess according to the capacities
  - Asteroids that are bigger should take larger portions of the overflow
  - We prefer if Vesta and Ceres take $1200/4400 = 3/11$ of the excess
  - We prefer if Hygiea and Pallas take $1000/4400 = 5/22$ of the excess
  - Capacities are expanded to handle the overflow (rounded up)

Asteroid	Vesta	Hygiea	Pallas	Ceres
Ideal # of People	1364	1136	1136	1364

$$1200 + \frac{3}{11}(600) = 1363.636363 \approx 1364$$

Ex: The Expanse

- Goal 3: Minimize overcrowding at each asteroid, proportionally allocating the excess among the asteroids
  - Constraints with deviational variables

$$
\begin{aligned}
x_{VV} + y_{VV} + x_{HV} + y_{HV} + x_{PV} + y_{PV} + x_{CV} + y_{CV} + d_6^- - d_6^+ &= 1364 \\
x_{VH} + y_{VH} + x_{HH} + y_{HH} + x_{PH} + y_{PH} + x_{CH} + y_{CH} + d_7^- - d_7^+ &= 1136 \\
x_{VP} + y_{VP} + x_{HP} + y_{HP} + x_{PP} + y_{PP} + x_{CP} + y_{CP} + d_8^- - d_8^+ &= 1136 \\
x_{VC} + y_{VC} + x_{HC} + y_{HC} + x_{PC} + y_{PC} + x_{CC} + y_{CC} + d_9^- - d_9^+ &= 1364
\end{aligned}
$$

  - Updated objective function for third priority

$$
\begin{aligned}
\text{Minimize} \quad & P_1(d_1^- + d_1^+ + d_2^- + d_2^+ + d_3^- + d_3^+ + d_4^- + d_4^+), \\
& P_2(d_5^+), \\
& P_3(d_6^- + d_6^+ + d_7^- + d_7^+ + d_8^- + d_8^+ + d_9^- + d_9^+)
\end{aligned}
$$

Ex: The Expanse

- Additional constraints
  - We cannot move more people than what is currently available

Asteroid	# of Martians	# of Earthers	*Capacity*
Vesta	1000	300	1200
Hygiea	450	800	1000
Pallas	1050	400	1000
Ceres	500	500	1200

  - List of constraints

$$x_{VV} + x_{VH} + x_{VP} + x_{VC} = 1000$$
$$y_{VV} + y_{VH} + y_{VP} + y_{VC} = 300$$
$$x_{HV} + x_{HH} + x_{HP} + x_{HC} = 450$$
$$y_{HV} + y_{HH} + y_{HP} + y_{HC} = 800$$
$$x_{PV} + x_{PH} + x_{PP} + x_{PC} = 1050$$
$$y_{PV} + y_{PH} + y_{PP} + y_{PC} = 400$$
$$x_{CV} + x_{CH} + x_{CP} + x_{CC} = 500$$
$$y_{CV} + y_{CH} + y_{CP} + y_{CC} = 500$$

Ex: The Expanse

- Additional integer constraints

$$x_{ij} \in \{0,1,\cdots\}$$
$$y_{ij} \in \{0,1,\cdots\}$$

- Download Expanse.xlsx from link Sheet 1 on course website
- Tab called Priority 1
  - Matrices of decision variables

Martians	Vesta	Hygiea	Pallas	Ceres	Total
Vesta	0	0	0	0	0
Hygiea	0	0	0	0	0
Pallas	0	0	0	0	0
Ceres	0	0	0	0	0
Total	0	0	0	0	

Earthers	Vesta	Hygiea	Pallas	Ceres	Total
Vesta	0	0	0	0	0
Hygiea	0	0	0	0	0
Pallas	0	0	0	0	0
Ceres	0	0	0	0	0
Total	0	0	0	0	

Deficit	Surplus
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0

Ex: The Expanse

- Tab called Priority 1
  - Notice all the different constraints and inspect formulas

Constraints:

	Deficit	Surplus	Computed	Constraint	Value	
0	0	0	0	=	0	*Balance at Vesta*
0	0	0	0	=	0	*Balance at Hygiea*
0	0	0	0	=	0	*Balance at Pallas*
0	0	0	0	=	0	*Balance at Ceres*
0	0	0	0	=	30000	*Total Distance Travelled*
0	0	0	0	=	1364	*Overcrowding at Vesta*
0	0	0	0	=	1136	*Overcrowding at Hygiea*
0	0	0	0	=	1136	*Overcrowding at Pallas*
0	0	0	0	=	1364	*Overcrowding at Ceres*
			0	=	1000	*Martians at Vesta*
			0	=	300	*Earthers at Vesta*
			0	=	450	*Martians at Hygiea*
			0	=	800	*Earthers at Hygiea*
			0	=	1050	*Martians at Pallas*
			0	=	400	*Earthers at Pallas*
			0	=	500	*Martians at Ceres*
			0	=	500	*Earthers at Ceres*

Ex: The Expanse

- Tab called Priority 1
  - First objective function

$$\text{Minimize} \qquad d_1^- + d_1^+ + d_2^- + d_2^+ + d_3^- + d_3^+ + d_4^- + d_4^+$$

  - Observe formula for objective function

39	**Objective function:**	
40	d1^- + d1^+ + d2^- + d2^+ + d3^- + d3^+ + d4^- + d4^+	
41	0	=SUM(B20:C23)

  - Q: What is “B20:C23” referring to and what is “SUM” doing?

  - Use Excel solver to find the optimal solution

Ex: The Expanse

- Tab called Priority 1
  - Optimal solution

Martians	Vesta	Hygiea	Pallas	Ceres	Total
Vesta	461	538	1	0	1000
Hygiea	0	140	65	245	450
Pallas	0	0	1050	0	1050
Ceres	1	0	0	499	500
Total	462	678	1116	744	

Earthers	Vesta	Hygiea	Pallas	Ceres	Total
Vesta	300	0	0	0	300
Hygiea	1	452	344	3	800
Pallas	0	0	400	0	400
Ceres	7	0	0	493	500
Total	308	452	744	496	

Deficit	Surplus
0	1.71E-13
0	0
0	0
0	0
0	152
594	0
6	0
0	724
124	0

Ex: The Expanse

- Tab called Priority 2
  - Notice the additional constraint and inspect formula

0	=		0	*First goal optimal*

  - Second objective function

Minimize $\quad d_5^+$

  - Observe formula for objective function

40	**Objective function:**	
41	d5^+	
42	0	=SUM(C24)

  - Use Excel solver to find optimal solution

Ex: The Expanse

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- Tab called Priority 2
  - Optimal solution

Martians	Vesta	Hygiea	Pallas	Ceres	Total
Vesta	306	442	1	251	1000
Hygiea	0	223	227	0	450
Pallas	0	6	1044	0	1050
Ceres	0	1	0	499	500
Total	306	672	1272	750	

Earthers	Vesta	Hygiea	Pallas	Ceres	Total
Vesta	204	0	96	0	300
Hygiea	0	448	352	0	800
Pallas	0	0	400	0	400
Ceres	0	0	0	500	500
Total	204	448	848	500	

Deficit	Surplus
0	1.279E-13
0	0
0	0
0	0
0	0
854	0
16	0
0	984
114	0

Ex: The Expanse

- Tab called Priority 3
  - Notice the additional constraint

0	=		0	*First goal optimal*
0	=		0	*Second goal optimal*

  - Second objective function

Minimize $d_6^- + d_6^+ + d_7^- + d_7^+ + d_8^- + d_8^+ + d_9^- + d_9^+$

  - Formula for this objective similar to first objective

41	**Objective function:**			
42	d6^- + d6^+ + d7^- + d7^+ + d8^- + d8^+ + d9^- + d9^+			
43	0			

  - Use Excel solver to find optimal solution

Ex: The Expanse

- Tab called Priority 3
  - Optimal solution

Martians	Vesta	Hygiea	Pallas	Ceres	Total
Vesta	643.77333	37.826667	0	318.4	1000
Hygiea	174.62667	275.37333	0	0	450
Pallas	0	368.4	681.6	0	1050
Ceres	0	0	0	500	500
Total	818.4	681.6	681.6	818.4	

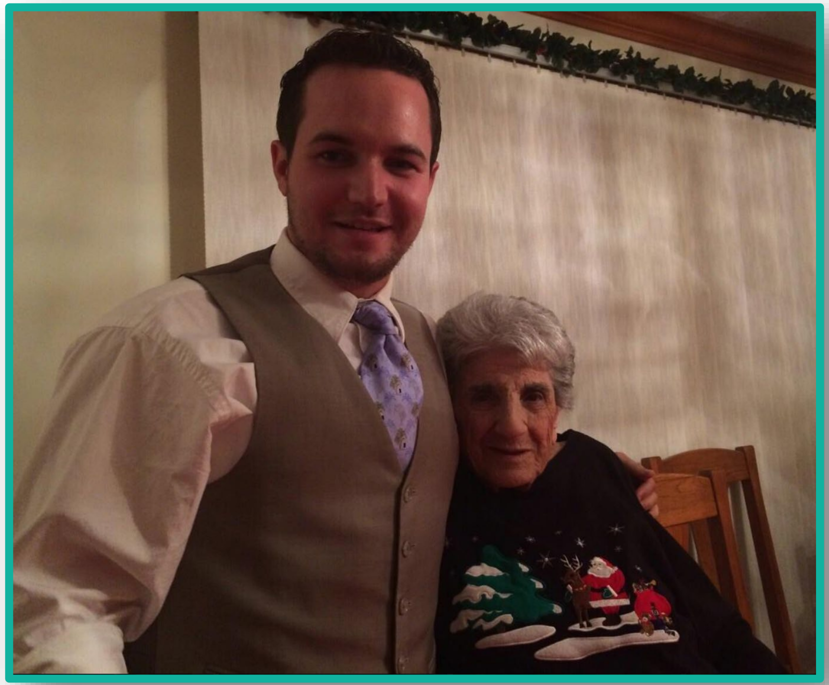
Earthers	Vesta	Hygiea	Pallas	Ceres	Total
Vesta	254.4	0	0	45.6	300
Hygiea	0	454.4	345.6	0	800
Pallas	291.2	0	108.8	0	400
Ceres	0	0	0	500	500
Total	545.6	454.4	454.4	545.6	

Deficit	Surplus
0	0
-1.28E-13	0
0	1.279E-13
0	0
0	0
0	0
0	0
0	1.421E-13
0	0

- Q: What is the problem with the optimal solution?

Ex: The Expanse

- Tab called Priority 3
  - Q: What do you mean you cannot move half a person?



  - Try to add integer constraints and see what happens

The End

Dale