Lecture 201

Produced by Dr. Worldwide

Welcome to the 305

Solving in Excel

- Algorithms for solving nonlinear programming models can be very complex
- Most algorithms can only guarantee that they find a local optimizer rather than a global one
- Excel Solver uses an algorithm called Generalized Reduced Gradient (GRG) to solve nonlinear problems
- This algorithm is designed to find a local optimizer within a certain “tolerance” level, and it can sometimes get “stuck”
- In problems not guaranteed to have a unique interior optimal solution, it is a good idea to run the GRG algorithm starting at several initial points

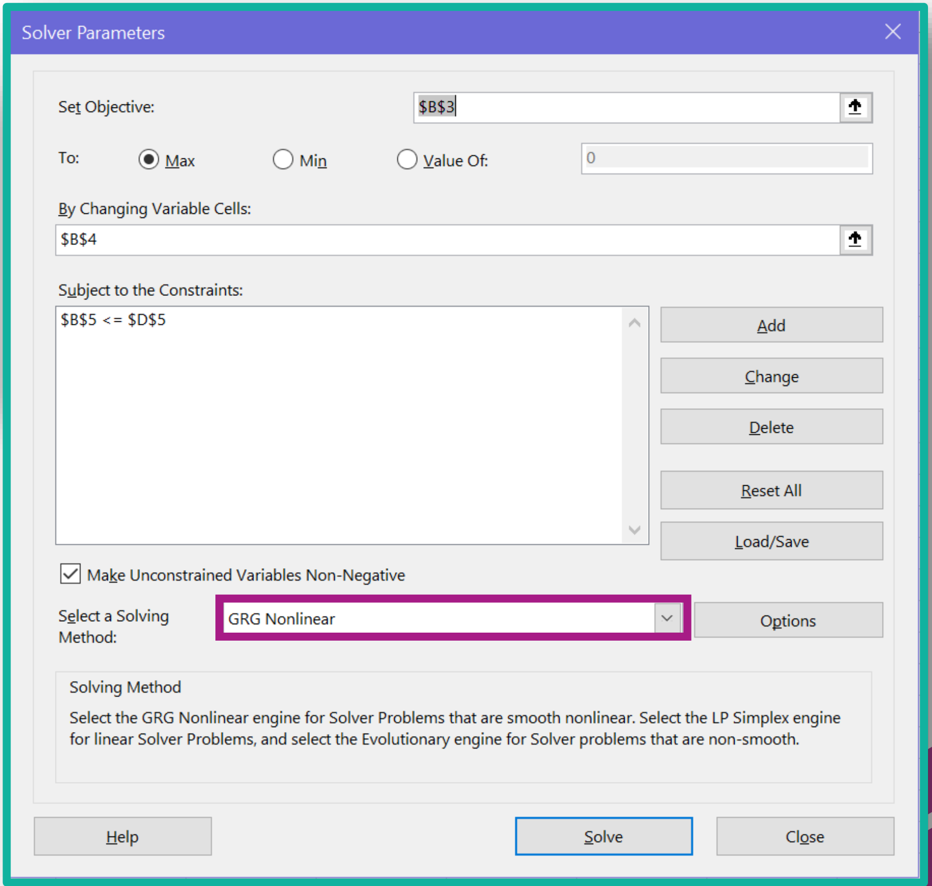
Solving in Excel

- Download NonlinearProfit.xlsx from link Sheet 1 on course website

- Inspect the spreadsheet and Solver

	A	B	C	D
1	**Maximizing nonlinear profit**			
2				
3	Profit:	7259.45366		
4	Variable (p):	34.4878049		
5	*Constraint*	34.4878049	<=	40

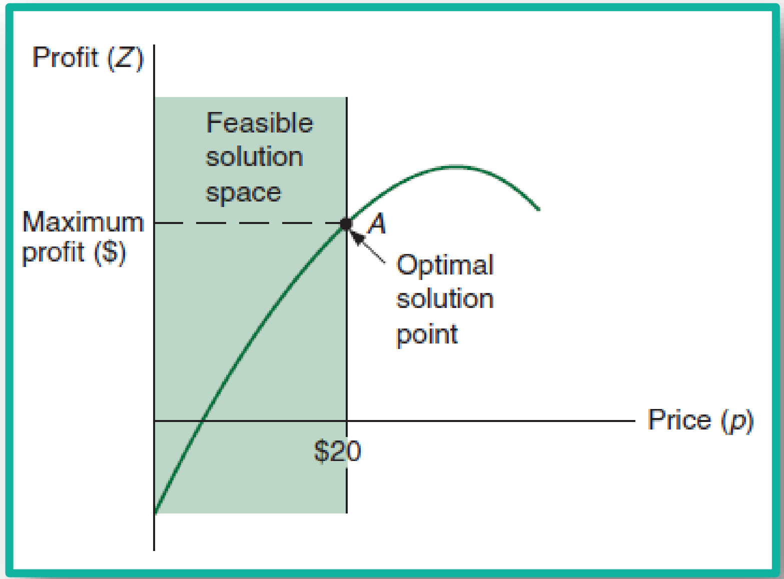
=1696.8*B4-24.6*B4^2-22000

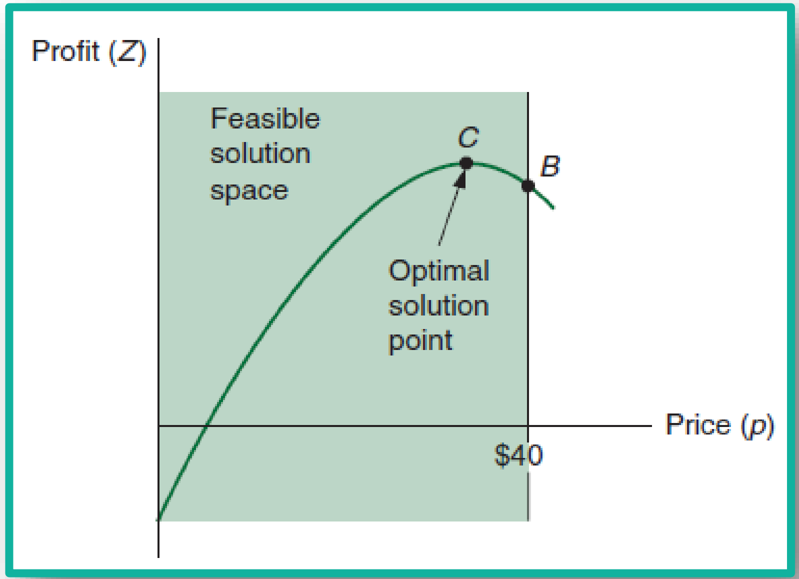


- Solution is for price ceiling of $40

Solving in Excel

- Q: What happens if you adjust the price ceiling to $20?
- Q: Is your answer consistent with what we have previously seen?





- Q: What happens if you completely drop the constraint?

Ex: Beaver Creek Pottery

- This company makes and sells clay bowls and clay mugs
- Model profit as a nonlinear function for maximization
- Examine the following relationships for the profit for bowls (x) and mugs (y)

$$Profit\ per\ Bowl = 4 - 0.1x$$
$$Profit\ per\ Mug = 5 - 0.2y$$

- Assume that there is only one constraint pertaining to labor

$$x + 2y = 40$$

- New optimization problem

Maximize	$(4 - 0.1x)x + (5 - 0.2\mathrm{y})\mathrm{y}$
Subject to	$x + 2y = 40$
	$x, y \geq 0$

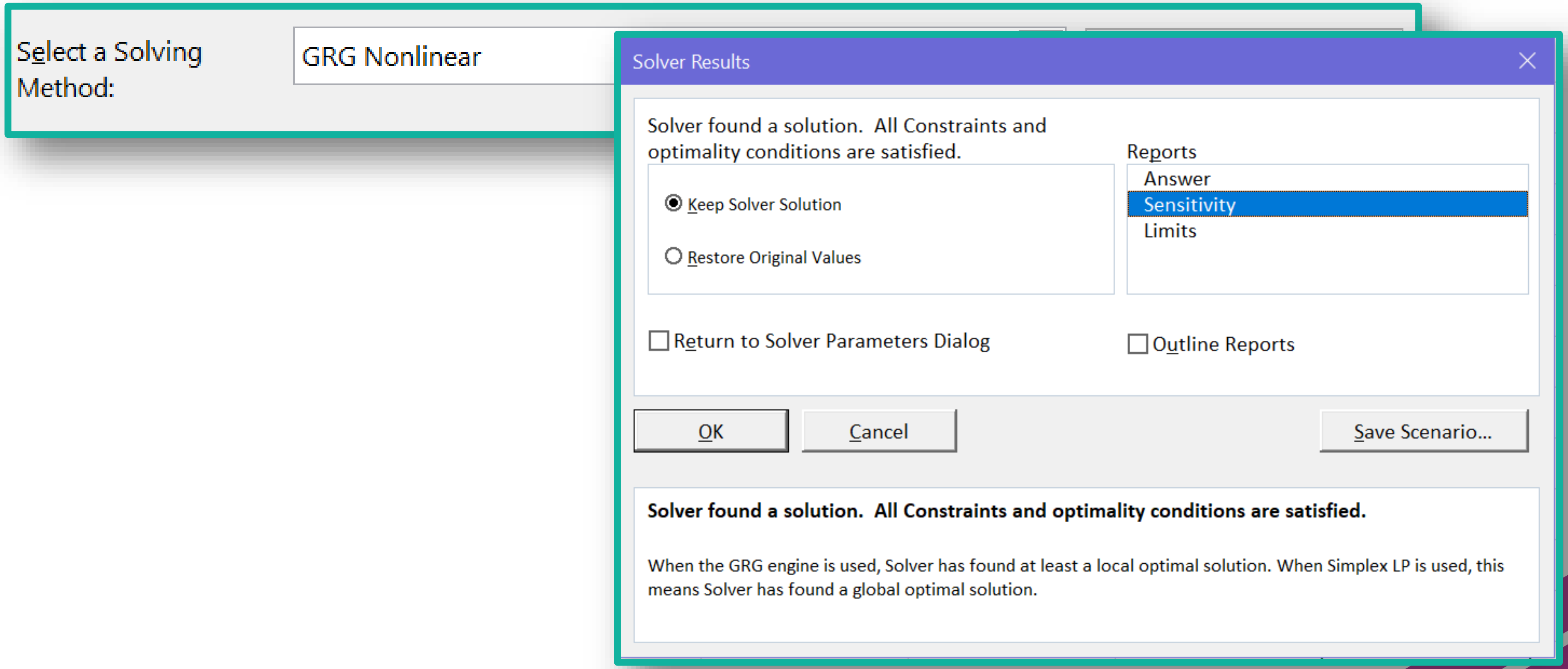
Ex: Beaver Creek Pottery

- Download BeaverCreekNonlinear.xlsx from link Sheet 2 on course website
- Inspect the spreadsheet and the nonlinear objective function

	A	B	C	D	E	F
1	**Beaver Creek Pottery Company (nonlinear)**					
2						
3	*Variables:*					
4	Bowls (x)	0				
5	Mugs (y)	0				
6						
7	Profit:	0	=(4-0.1*B4)*B4+(5-0.2*B5)*B5			
8						
9	*Constraint*	x	y	Used	Constraint	Allowed
10	Labor	1	2	0	=	40

Ex: Beaver Creek Pottery

- Run Excel Solver using algorithm Simplex LP and observe what happens
- Run Excel Solver using GRG Nonlinear and select the sensitivity report



Ex: Beaver Creek Pottery

- Solution is to produce 18.3 bowls and 10.8 mugs for a profit of $70.42

- Inspect the sensitivity report

Variable Cells

Cell	Name	Final Value	Reduced Gradient
B4	Bowls (x)	18.33333327	0
B5	Mugs (y)	10.83333337	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
D10	Labor Used	40	0.333332151

- The Lagrange multiplier is analogous to the shadow price from before

Ex: Western Clothing

- Company produces two types of jeans
  - Designer
  - Straight-leg

- Demand for designer jeans (x_1) and demand for straight-leg jeans (x_2) are functions of the corresponding prices, and follow the relations:

$$x_1 = 1500 - 24.6p_1$$
$$x_2 = 2700 - 63.8p_2$$

- Designer jeans cost $12 per pair and straight-leg jeans cost $9 per pair

- Each pair of jeans requires the following:

	Cloth (yd)	Cutting time (min)	Sewing time (min)
Designer	2	3.6	7.2
Straight-leg	2.7	2.9	8.5

Ex: Western Clothing

- Company has the following capacities
  - 6,000 yards of cloth
  - 8,500 minutes of cutting time
  - 15,000 minutes of sewing time

- Decision variables
  - $x_1 = \text{Number of designer jeans to produce}$
  - $x_2 = \text{Number of straight} - \text{leg jeans to produce}$

- Objective function for profit (revenue-cost)

$$Z = (p_1 x_1 + p_2 x_2) - (12x_1 + 9x_2) = (p_1 - 12)x_1 + (p_2 - 9)x_2$$

- Goal is to find out how many jeans to produce so we need to use the relationships between the price and the number of each type to produce

Ex: Western Clothing

- Updated objective function for profit

$$Z = (p_1 - 12)x_1 + (p_2 - 9)x_2$$
$$= \left(\frac{1500 - x_1}{24.6} - 12\right) x_1 + \left(\frac{2700 - x_2}{63.8} - 9\right) x_2$$

- Constraints based on limited resources

$2x_1 + 2.7x_2 \leq 6{,}000$ (Cloth)
$3.6x_1 + 2.9x_2 \leq 8{,}500$ (Cutting Time)
$7.2x_1 + 8.5x_2 \leq 15{,}000$ (Sewing Time)

- Nonnegativity constraints

$x_1 \geq 0$ *and* integer (if possible)
$x_2 \geq 0$ *and* integer (if possible)

- Download WesternClothing.xlsx from link Sheet 3 on course website

Ex: Western Clothing

- Run Excel Solver

Variable Cells

Cell	Name	Final Value	Reduced Gradient
B4	Designer jeans (x1)	602.3995467	0
B5	Straight-leg jeans (x2)	1062.900112	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
D10	Cloth Used	4074.629395	0
D11	Cutting time Used	5251.048693	0
D12	Sewing time Used	13371.92769	0

- Add integer constraints and run Excel Solver (Sensitivity not available)

Ex: Facility Location

- Clayton County Rescue Squad and Ambulance Service wants to build a centralized facility to service five rural towns
  - Abbeville
  - Benton
  - Clayton
  - Dunning
  - Eden

- Let (x, y) denote the location of the proposed facility

- Let (x_i, y_i) denote the location of town i

- Distance between the proposed facility and a town i

$$d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}$$

Ex: Facility Location

- Town locations and number of annual trips are given below

Town	Coordinates x_i	y_i	Annual trips (t_i)
Abbeville	20	20	75
Benton	10	35	105
Clayton	25	9	135
Dunning	32	15	60
Eden	10	8	90

- Two ideas to consider
  - The facility should be placed closed to the center of all these towns
  - The facility should be placed closed to towns that are visited more often

Ex: Facility Location

- Q: In which location should we place the facility that minimizes the distance to each of the towns, prioritizing those that are visited more often?

- Nonlinear program (Unconstrained or constrained)

  Minimize $\quad \sum d_i t_i = \sum t_i \sqrt{(x_i - x)^2 + (y_i - y)^2}$

  Subject to $\quad x, y \geq 0 \longrightarrow$ Is this necessary?

- Download FacilityLocation.xlsx from link Sheet 4 on course website

- Run Excel Solver both with and without positive constraint

- Q: Did going from constrained to unconstrained get you an error?

The End

Dale