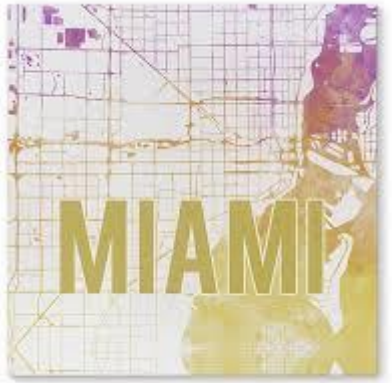

# Lecture 22

Produced by Dr. Worldwide

*Welcome to the 305*

# Ex: Admissions at State




- State has increased its tuition for all students in each of the last 5 years
- University administration always thought the number of applications received was independent of tuition
- Drops in applications and enrollment prove this idea to be wrong
- University admissions officials developed the following relationships between the number of applicants ($x_i$) and cost of tuition ($t_i$)

  $x_1 = 21000 - 12t_1$ (Relationship for in-state applicants)
  $x_2 = 35000 - 6t_2$ (Relationship for out-of-state applicants)

- University desires to develop a planning model to indicate the in-state and out-of-state tuitions, as well as, the number of students that could be expected to enroll in the freshman class

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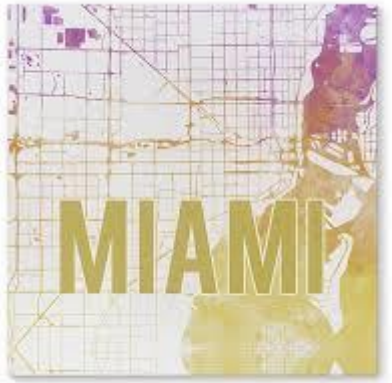


- Constraints based on resources
  - Not enough classroom space for more than 1,400 students
  - Needs at least 700 freshmen to meet all its class size objectives
  - At most 800 dorm rooms available for freshmen

- Historical expectations
  - 55% of all in-state freshmen desire to live in dorms
  - 72% of all out-of-state freshmen desire to live in dorms

- Uphold the academic standards of the institution
  - Average SAT is 960 for in-state students
  - Average SAT is 1150 for out-of-state students
  - University wants the entering freshmen to average 1,000

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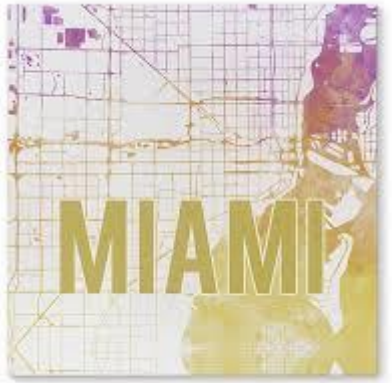


- Legislative requirements
  - State is supported by the state LOL ☺
  - The legislature wants to make sure that State doesn't just admit out-of-state students because they pay more money or have better SAT scores
  - Policy that no more than 55% of the entering freshman can be out-of-state students

- Q: How much should State charge, what would the total tuition be, and how many in-state and out-of-state students should they expect?

- Decision variables
  - We have a choice between $x_1$ and $x_2$ or $t_1$ and $t_2$
  - Related through the following equations

$$x_1 = 21000 - 12t_1$$
$$x_2 = 35000 - 6t_2$$

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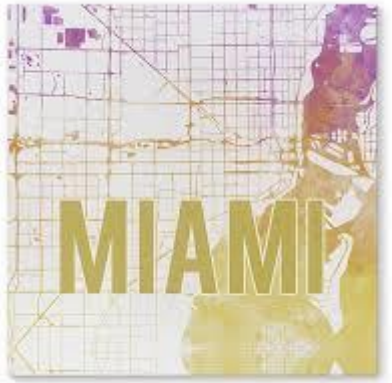


- Objective function
  - Goal is to maximize the revenue in tuition
  - Total tuition based off in-state and out-of-state students

$$x_1 t_1 + x_2 t_2 = x_1 \times \frac{(21000 - x_1)}{12} + \mathrm{x}_2 \times \frac{(35000 - x_2)}{6}$$

- Constraints
  - Maximum number of freshmen

    $x_1 + x_2 \leq 1400$

  - Minimum number of freshmen

    $x_1 + x_2 \geq 700$

  - Maximum number of dormitories

    $0.55x_1 + 0.72x_2 \leq 800$

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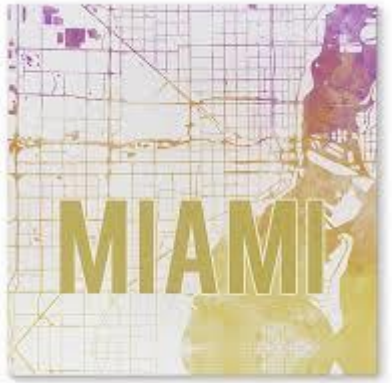


- Constraints
  - Average SAT Scores

$$\frac{960x_1 + 1150x_2}{x_1 + x_2} \geq 1000$$

$$(960 - 1000)x_1 + (1150 - 1000)x_2 = -40x_1 + 150x_2 \geq 0$$

  - Maximum for out-of-state students

$$\frac{x_2}{x_1 + x_2} \leq 0.55$$

$$-0.55x_1 + (1 - 0.55)x_2 = -0.55x_1 + 0.45x_2 \leq 0$$

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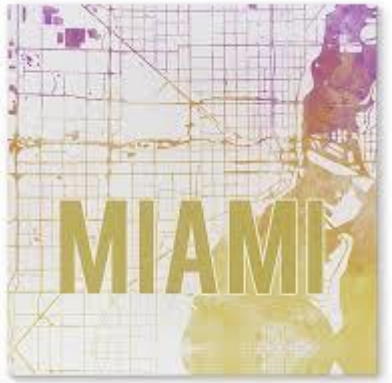


- Nonlinear program

Maximize $x_1 \times \frac{(21000-x_1)}{12} + x_2 \times \frac{(35000-x_2)}{6}$

Subject to
$$x_1 + x_2 \leq 1400$$
$$-x_1 - x_2 \leq -700$$
$$0.55x_1 + 0.72x_2 \leq 800$$
$$40x_1 - 150x_2 \leq 0$$
$$-0.55x_1 + 0.45x_2 \leq 0$$
$$x_1, x_2 \geq 0$$

- Download Admissions.xlsx from link Sheet 1 on course website

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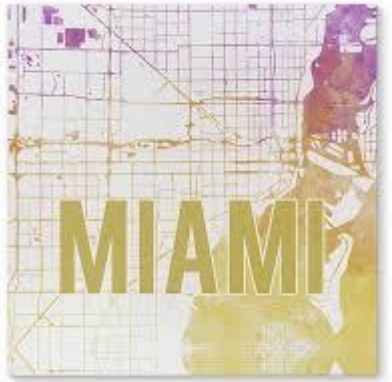
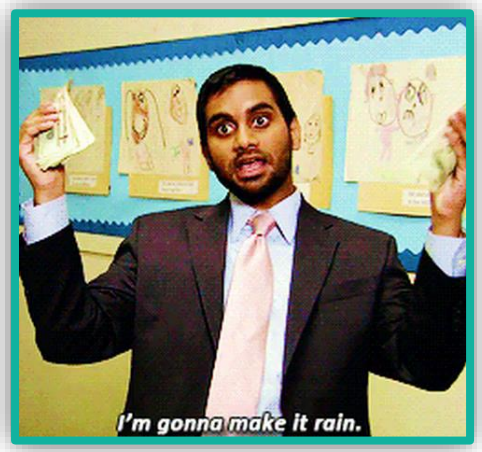



- Use Excel to find the optimal solution is $x_1 = 559.44$ and $x_2 = 683.76$
- Consequently, we find the optimal tuition for in-state and out-of-state students

$$t_1 = \frac{21000 - x_1}{12} = \frac{21000 - 559.44}{12} = \$1{,}703.38$$

$$t_2 = \frac{35000 - x_2}{6} = \frac{35000 - 683.76}{6} = \$5{,}719.37$$

- Expected total tuition from freshmen is $x_1 t_1 + x_2 t_2 = \$4{,}863{,}622.37$



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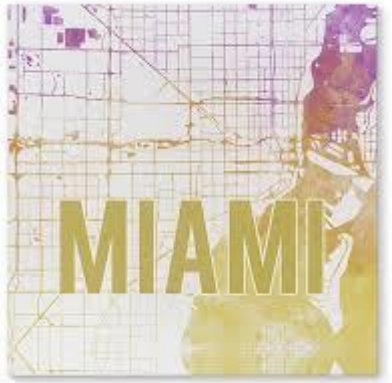
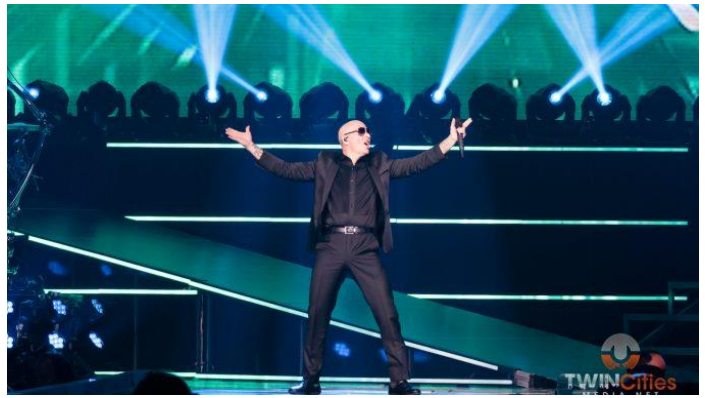


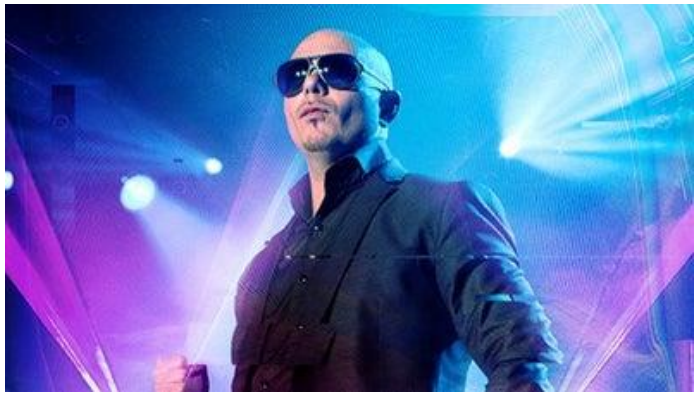

- Solve again in Excel and save sensitivity report

Constraints

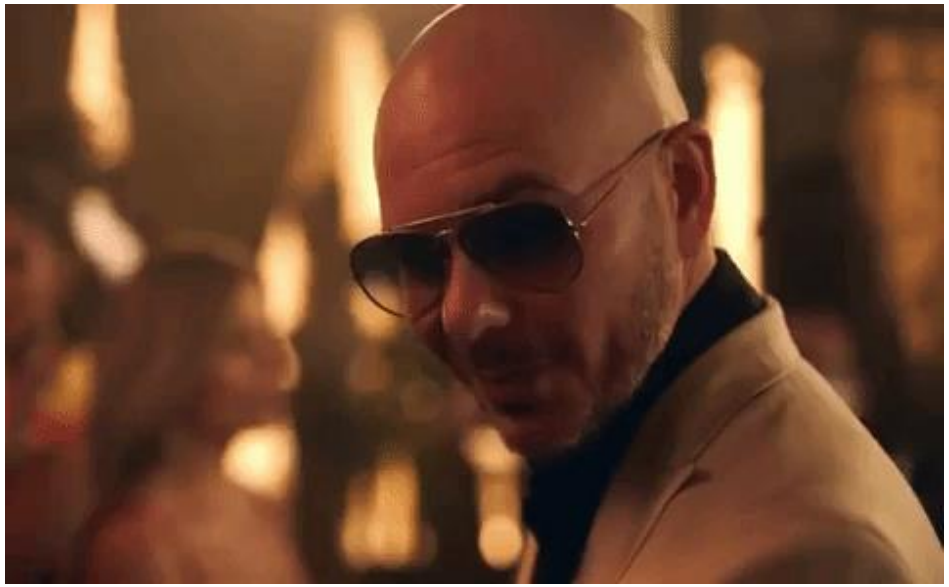
| Cell | Name | Final Value | Lagrange Multiplier |
|---|---|---|---|
| $D$5 | Max. students Computed | 1243.201243 | 0 |
| $D$6 | Min. students Computed | -1243.201243 | 0 |
| $D$7 | Dormitory capacity Computed | 800 | 5949.524432 |
| $D$8 | SAT average Computed | -80186.48019 | 0 |
| $D$9 | Max. out-of-state Computed | 0 | 2937.233899 |

- Q: What does it mean that when the Lagrange multiplier is $0$?



# The End



Dale