Lecture 4

Produced by Dr. Worldwide

Welcome to the 305

Special Cases

- Infeasible problem
  - A linear program is infeasible if there is no point that satisfies all the constraints
  - Consider the linear program

Maximize	$5x + 3y$
Subject to	$4x + 2y \leq 8$
	$x \geq 4$
	$\mathrm{y} \geq 6$

  - When $x \geq 4$ and $\mathrm{y} \geq 6$, $4x + 2y > 8$

$$4(4) + 2(6) = 28 > 8$$

$$4(5) + 2(7) = 34 > 8$$

Special Cases

- Multiple optimal solutions
  - A linear program may have multiple optimal solutions if there are two or more extreme points along the optimal level curve for the problem
  - Consider the linear program

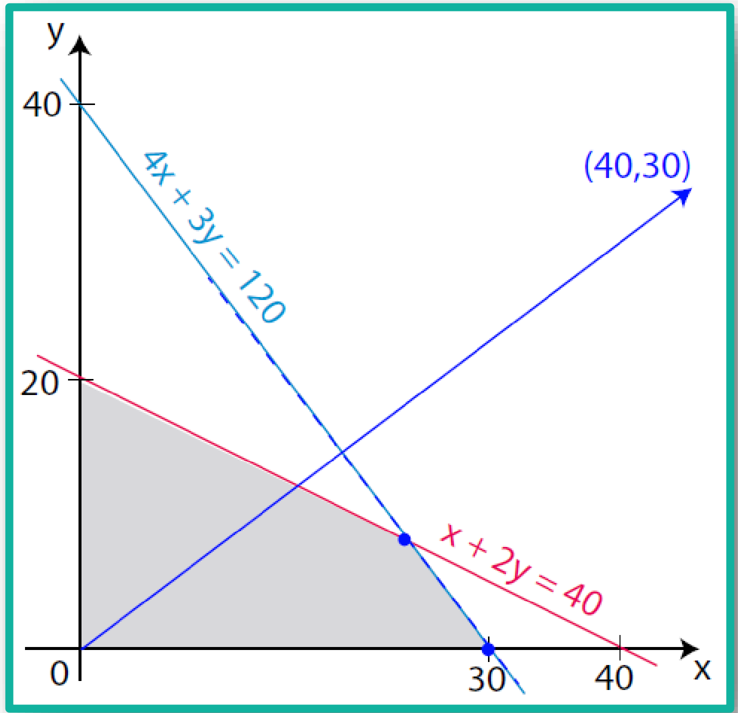
Maximize $40x + 30y$

Subject to $x + 2y \leq 40$

$4x + 3y \leq 120$

$x \geq 0$

$y \geq 0$



  - Optimal points: (24,8) & (30,0)

$$40(24) + 30(8) = 1200$$
$$40(30) + 30(0) = 1200$$

Special Cases

- Unbounded problem
  - A linear program is unbounded if the feasible region is not closed and the objective function grows (decreases) indefinitely without bound
  - Similar to the infeasible problem where no solution exists
  - Consider the two linear programs with identical feasible regions

A) Maximize $2x + y$

Subject to
$x + y \geq 5$
$x \leq 4$
$\mathrm{x} \geq 0$
$\mathrm{y} \geq 0$

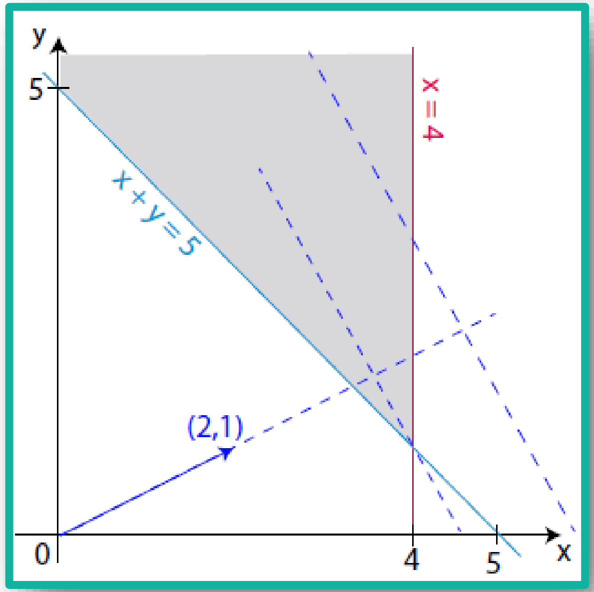
B) Minimize $2x + y$

Subject to
$x + y \geq 5$
$x \leq 4$
$\mathrm{x} \geq 0$
$\mathrm{y} \geq 0$

- Which linear program is unbounded, A or B?

Special Cases

- Unbounded problem
  - In both linear programs, the feasible region is unbounded



  - The maximization linear program is unbounded
  - The minimization linear program has a single optimal solution at (0,5)

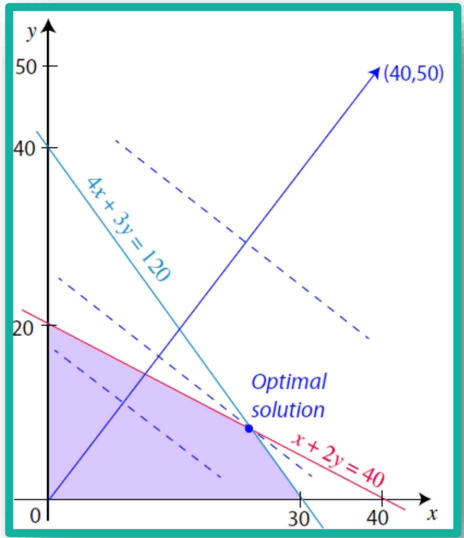
Computer Solution

- Majority of linear programs solved using a computer
- Excel's built-in tool called Solver is capable of handling linear optimization using the simplex algorithm from George Dantzig
- Recall: Beaver Creek Pottery Company from Lecture 2
  - Download BeaverCreek.xlsx from website link called Sheet 1
  - Linear Program

Maximize $40x + 50y$

Subject to

$$x + 2y \le 40$$
$$4x + 3y \le 120$$
$$x \ge 0$$
$$y \ge 0$$



  - Optimal: 24 Bowls and 8 Mugs

Computer Solution

- Preview of Spreadsheet

	A	B	C	D	E	F	G
1	**The Beaver Creek Pottery Company**						
2							
3	Products:	Bowl	Mug				
4	Profit per unit:	40	50				
5	Resources:			Usage	Constraint	Available	Left over
6	Labor (hr/unit)	1	2	0	<=	40	40
7	Clay (lb/unit)	4	3	0	<=	120	120
8							
9							
10	Production:						
11	Bowls =	0					
12	Mugs =	0					
13	Profit =	0					

Computer Solution

- Preview of Spreadsheet

	A	B	C	D	E	F	G
1	**The Beaver Creek Pottery Company**						
2							
3	Products:	Bowl	Mug				
4	Profit per unit:	40	50				
5	Resources:			Usage	Constraint	Available	Left over
6	Labor (hr/unit)	1	2	0	<=	40	40
7	Clay (lb/unit)	4	3	0	<=	120	120
8							
9							
10	Production:						
11	Bowls =	0					
12	Mugs =	0					
13	Profit =	0					

Objective Function: B4*B*11+C4*B12

Computer Solution

- Preview of Spreadsheet

	A	B	C	D	E	F	G
1	**The Beaver Creek Pottery Company**						
2							
3	Products:	Bowl	Mug				
4	Profit per unit:	40	50	Labor Used: B6*B11+C6*B12			
5	Resources:			Usage	Constraint	Available	Left over
6	Labor (hr/unit)	1	2	0	<=	40	40
7	Clay (lb/unit)	4	3	0	<=	120	120
8				Clay Used: B7*B11+C7*B12			
9							
10	Production:						
11	Bowls =	0					
12	Mugs =	0					
13	Profit =	0					

Computer Solution

- Preview of Spreadsheet

	A	B	C	D	E	F	G
1	**The Beaver Creek Pottery Company**						
2							
3	Products:	Bowl	Mug				
4	Profit per unit:	40	50				
5	Resources:			Usage	Constraint	Available	Left over
6	Labor (hr/unit)	1	2	0	<=	40	40
7	Clay (lb/unit)	4	3	0	<=	120	120
8						Capacity	
9							
10	Production:						
11	Bowls =	0					
12	Mugs =	0					
13	Profit =	0					

Computer Solution

- Preview of Spreadsheet

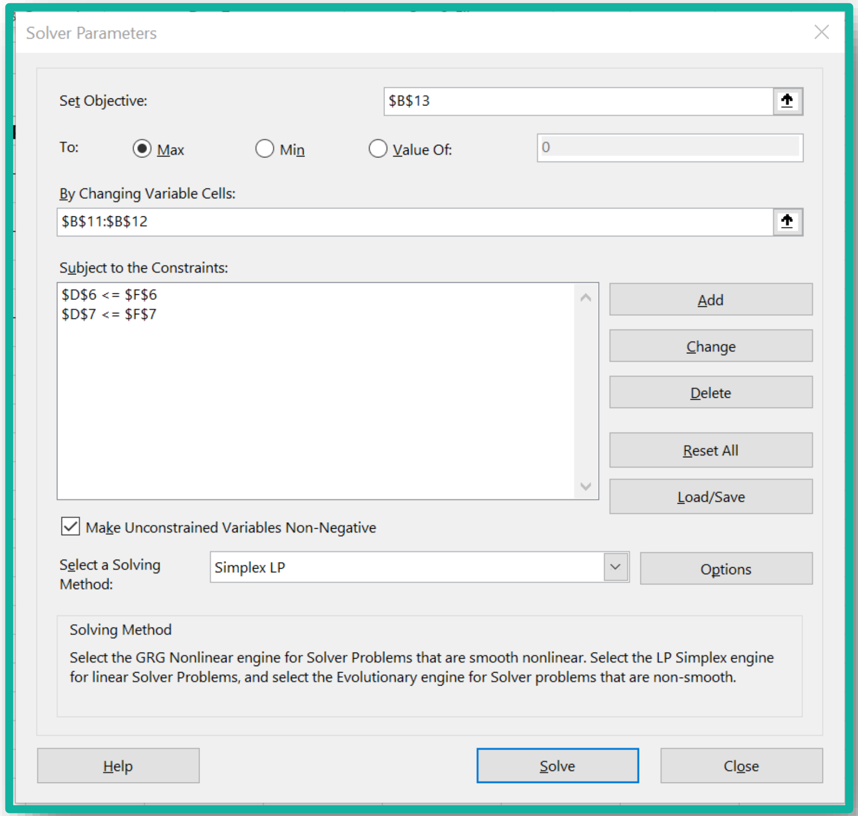
	A	B	C	D	E	F	G
1	**The Beaver Creek Pottery Company**						
2							
3	Products:	Bowl	Mug				
4	Profit per unit:	40	50				
5	Resources:			Usage	Constraint	Available	Left over
6	Labor (hr/unit)	1	2	0	<=	40	40
7	Clay (lb/unit)	4	3	0	<=	120	120
8							
9							
10	Production:						
11	Bowls =	0					
12	Mugs =	0					
13	Profit =	0					

Labor Waste: F6-D6

Clay Waste: F6-D6

Computer Solution

- Optimizing with Excel
  - Select Data and then select Solver by the Analyze section
  - Observe window



Computer Solution

- Optimizing with Excel
  - Cell you are trying to optimize with objective function

Set Objective: $B$13

  - Maximize or minimize

To: (•) Max ( ) Min

  - Choose your decision variables

10	Production:	
11	Bowls =	0
12	Mugs =	0

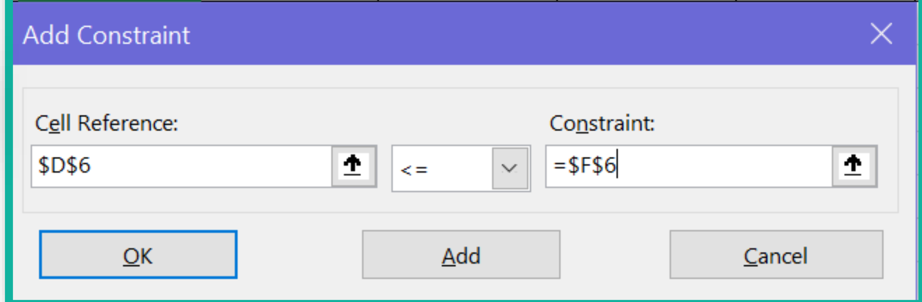
By Changing Variable Cells: $B$11:$B$12

Computer Solution

- Optimizing with Excel
  - Create your constraints using Add
  - You can type or click to select cell
  - You will see your constraints in the Subject to the Constraints
  - Notice box for nonnegativity

☑ Make Unconstrained Variables Non-Negative

B	C	D	E	F
ottery Company				
Bowl	Mug			
40	50			
		Usage	Constraint	Available
1	2	0	<=	40
4	3	0	<=	120



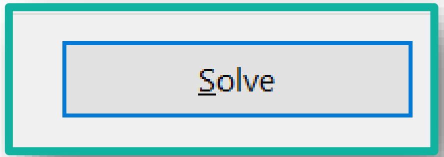
5	Resources:			Usage	Constraint	Available
6	Labor (hr/unit)	1	2	0	<=	40
7	Clay (lb/unit)	4	3	0	<=	120

Subject to the C

$D$6 <= $F$6
$D$7 <= $F$7

Computer Solution

- Optimizing with Excel
  - Select Solve



- Optimal solution can be found in decision variables

Production:	
Bowls =	0
Mugs =	0
Profit =	0

Production:	
Bowls =	24
Mugs =	8
Profit =	1360

Ex: *Annabelle Invests*

- Annabelle Sizemore has a massive amount of money (AKA stacks) from numerous sources that she needs to do something with (AKA make it rain)

- After researching the market, she has decided to split her money to 2 places
  - S&P index fund from Shield Securities
  - Internet stock fund from Madison Funds, Inc.

- Q: How should Annabelle split her money in these two funds?

- Decision Variables
  - $x = Amount\ Invested\ in\ S\&P\ Index\ Fund$
  - $y = Amount\ Invested\ in\ Internet\ Stock\ Fund$
  - $u = Number\ of\ S\&P\ Index\ Fund\ Shares$
  - $v = Number\ of\ Internet\ Stock\ Fund\ Shares$

Ex: *Annabelle Invests*

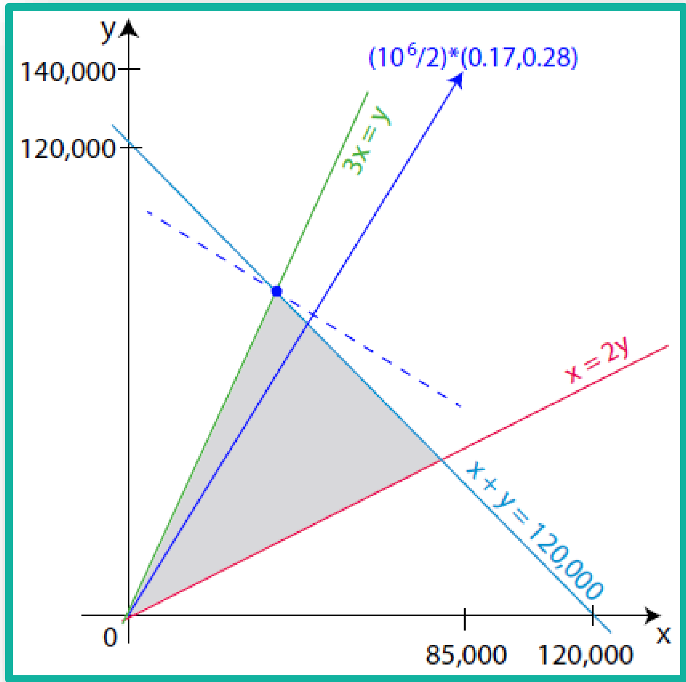
- Average annual return over the last 3 years for the S&P index fund was 17% and 28% for the internet stock fund
  - Goal: Maximize return on her investment
  - Objective function: $Z = 0.17x + 0.28y$
  - Price per share of S&P index fund is $175 and $208 for internet stock fund
  - Objective function: $Z = 0.17(175u) + 0.28(208v)$

- Constraints
  - Must invest nonnegative amounts : $x \geq 0 \;\&\; y \geq 0 \quad (u \geq 0 \;\&\; v \geq 0)$
  - Only has $120,000 to invest: $x + y \leq 120{,}000 \quad (175u + 208v \leq 120{,}000)$
  - The proportion of the dollar amount she invests in the index fund relative to the internet fund should be at least one-third: $x/y \geq 1/3$ or $3x - y \geq 0$ $(3(175u) - 208v \geq 0)$
  - Amount invested in index fund no more than twice the amount invested in the internet fund: $x \leq 2y$ or $x - 2y \geq 0 \quad (175u - 2(208v) \geq 0)$

Ex: *Annabelle Invests*

- Full linear program

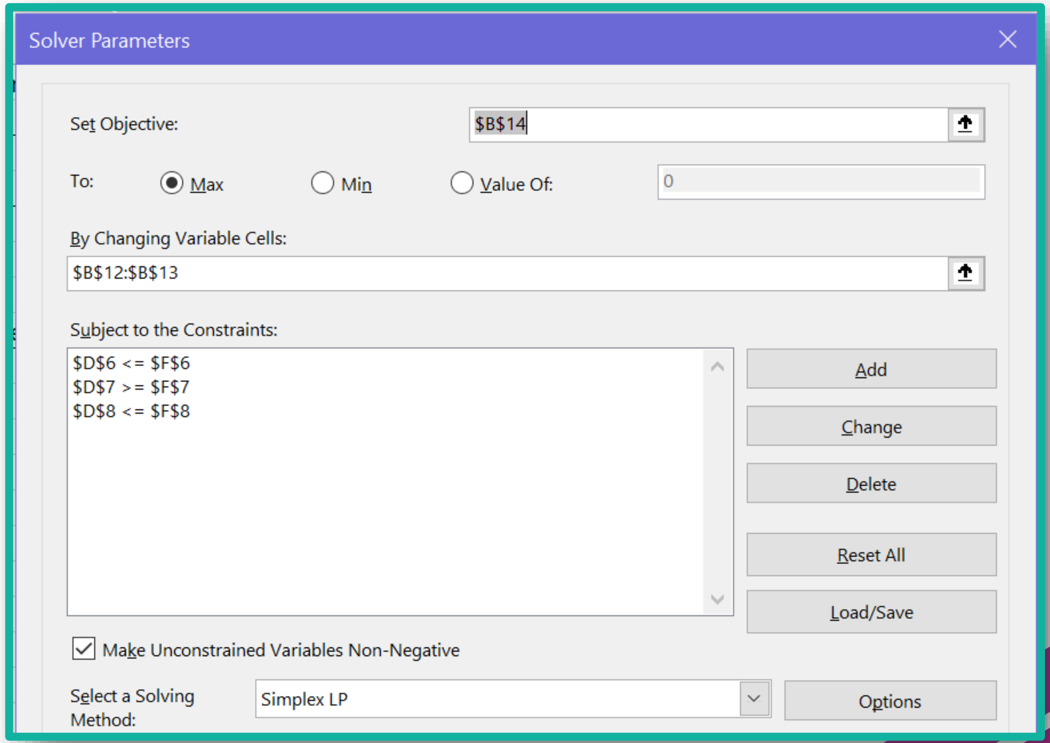
Maximize	$0.17x + 0.28y$
Subject to	$x + y \leq 120{,}000$
	$3x - y \geq 0$
	$x - 2y \leq 0$
	$x \geq 0$
	$y \geq 0$



Maximize	$0.17(175u) + 0.28(208v)$
Subject to	$175u + 208v \leq 120{,}000$
	$3(175u) - 208v \geq 0$
	$175u - 2(208v) \geq 0$
	$u \geq 0$
	$v \geq 0$

Ex: *Annabelle Invests*

- Alternative approach: use growth vector and level curves (contours)
  - Download AnnabelleInvest.xlsx from website link called Sheet 2
  - Try to use Excel Solver to find the optimal solution
  - Solution
    - x =\$30,000
    - y =\$90,000
  - Return is \$30,300



The End

Dale