Lecture 5

Produced by Dr. Worldwide

Welcome to the 305

Sensitivity Analysis

- A sensitivity analysis is how we investigate the effect changes in the objective function and constraints have on the optimal solution

- Types of changes
  - Changes in the objective function coefficients
  - Changes in the constraint quantity values
  - Changes in the constraint coefficients
  - Additional constraints
  - Additional decision variables

- Excel's Solver can handle changes in the first two types

- Other types involve rerunning Excel's Solver with different information

Changing Objective Function

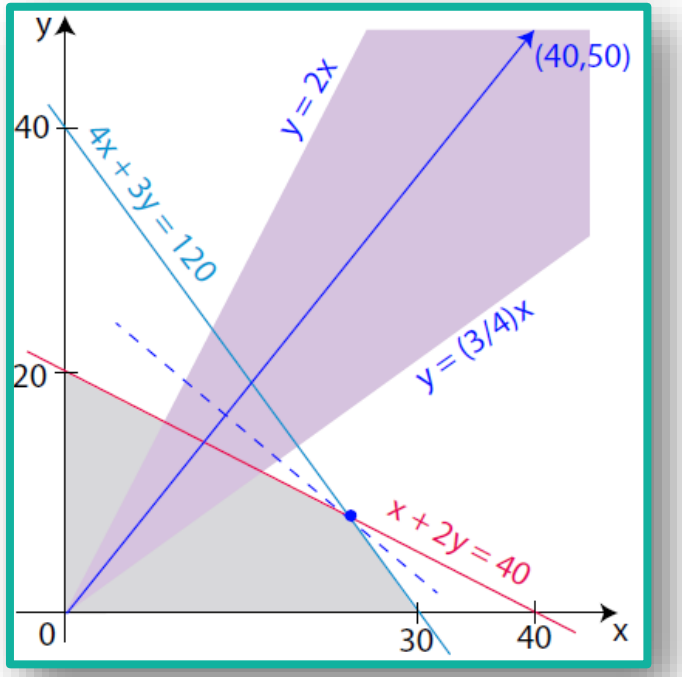
- Q: How much can parameters change without changing the optimal solution?
- Changes in objective function parameters lead to changes in the direction of level curves in a graph
- Consider the Beaver Creek linear program

$$\begin{aligned} \text{Maximize} \quad & 40x + 50y \\ \text{Subject to} \quad & x + 2y \leq 40 \\ & 4x + 3y \leq 120 \\ & x \geq 0 \\ & y \geq 0 \end{aligned}$$

- Recall optimal solution was $(24,8)$

Changing Objective Function

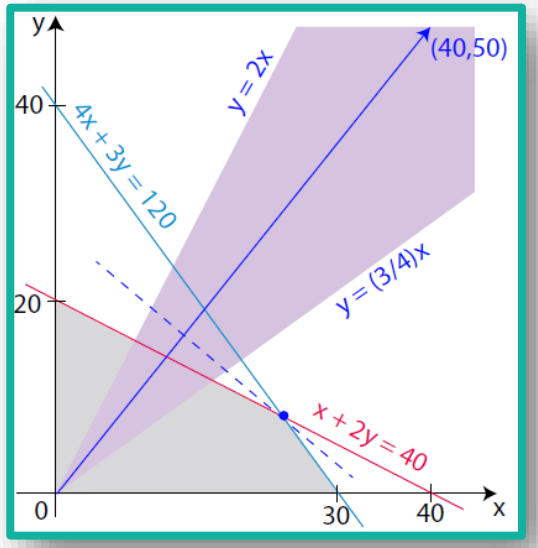
- We will "tilt" the objective function coefficients $(a, b) = (40,50)$ until the optimal solution changes



- Any vector in direction of (a, b), where (a, b) is in the purple region will lead to the same optimal solution

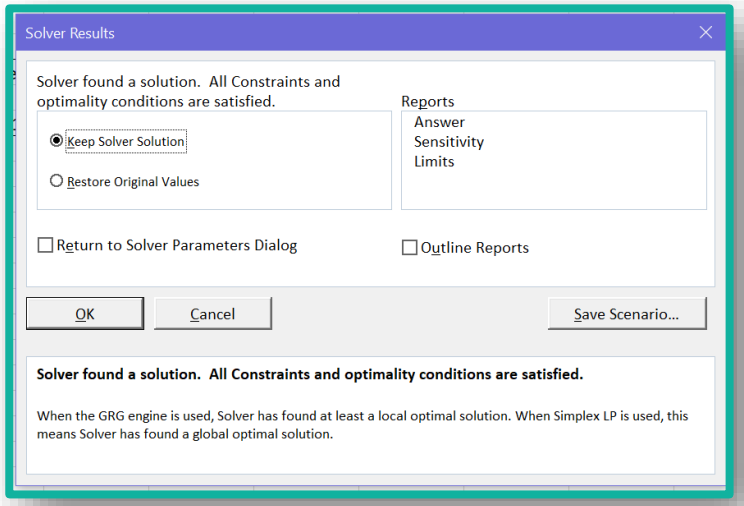
Changing Objective Function

- Edges of purple cone are perpendicular to constraints
  - Line $y = 2x$ is perpendicular to $x + 2y = 40$ $\left(y = 20 - \frac{1}{2}x\right)$
  - Line $y = \frac{3}{4}x$ is perpendicular to $4x + 3y = 120$ $\left(y = 40 - \frac{4}{3}x\right)$
- Points in the purple cone are characterized by slopes
- Points (a, b) in purple cone must satisfy $\frac{3}{4} \leq \frac{b}{a} \leq 2$
- If we fix $b = 50$, then $25 \leq a \leq \frac{200}{3}$
- If we fix $a = 40$, then $30 \leq b \leq 80$

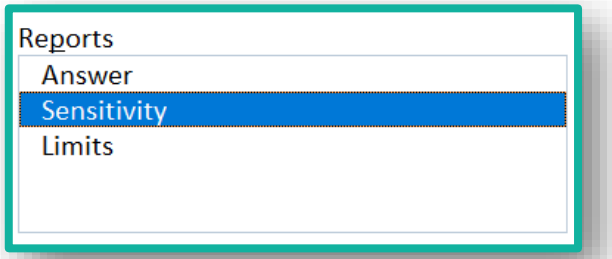


Ex: Beaver Creek Pottery

- Download BeaverCreek.xlsx from website link called Sheet 1
- We start by considering changes to the objective function $40x + 50y$
- Attempt to solve linear program produces menu of options



Select Sensitivity



- Selecting OK creates a new sheet in Excel file

Ex: Beaver Creek Pottery

- Sensitivity analysis for objective function displayed below

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B11	Bowls = Bowl	24	0	40	26.66666667	15
B12	Mugs = Bowl	8	0	50	30	20

- If we fix $b = 50$, then $25 = 40 - 15 \leq a \leq 40 + 26.67 = \frac{200}{3}$
- If we fix a $= 40$, then $30 = 50 - 20 \leq b \leq 50 + 30 = 80$
- Next, we consider changes to the constraint quantities

Changing Constraint Quantity

- Changing constraint quantities (c, d) from $(40,120)$

  Subject to
  $$x + 2y \leq c$$
  $$4x + 3y \leq d$$
  $$x \geq 0$$
  $$y \geq 0$$

- Changes in these quantities cause vertical shifts of constraints in graph
- Constraint is binding at the optimal solution if the constraint holds with equality
- Constraint is non-binding at optimal solution if the constraint fails with equality
- In this sensitivity analysis, there is not concern about different optimal solutions

Changing Constraint Quantity

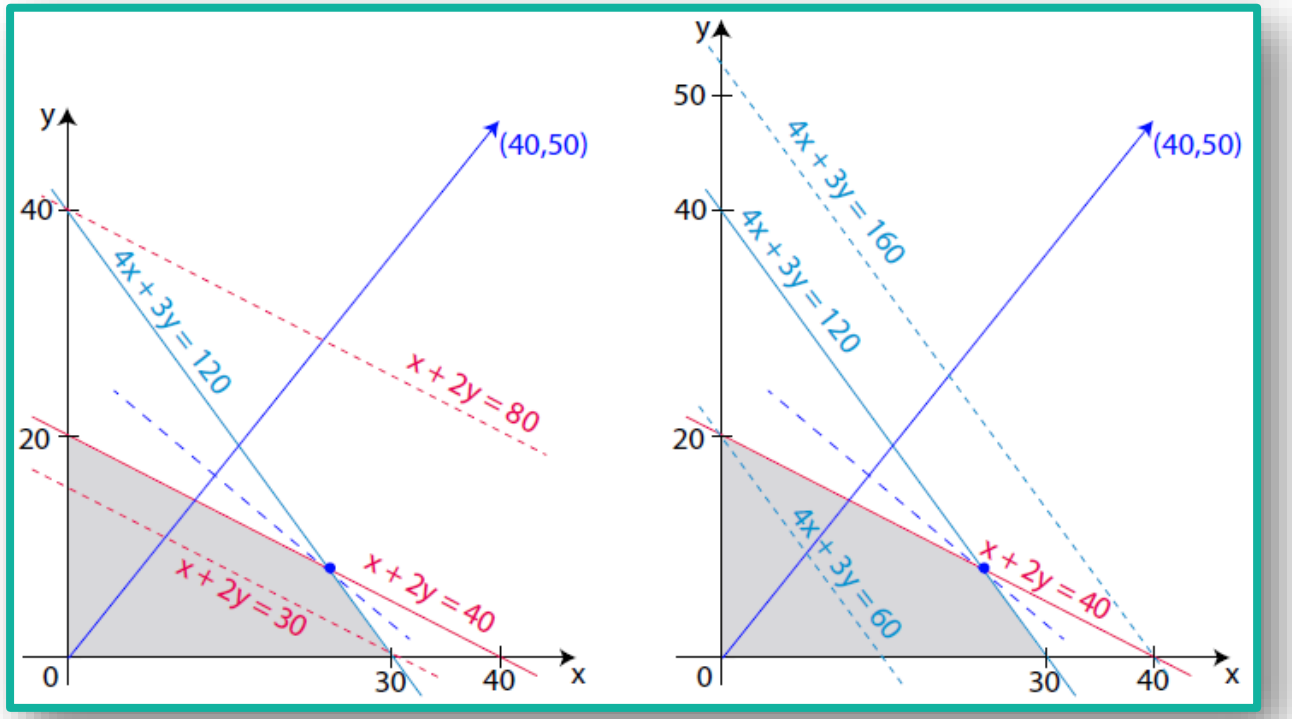
- Binding constraints at the optimal solution $(24,8)$
  - $x + 2y = 24 + 2(8) = 40 = \mathrm{c}$
  - $4x + 3y = 4(24) + 3(8) = 120 = \mathrm{d}$

- Non-binding constraints at the optimal solution $(24,8)$
  - $x = 24 \neq 0$
  - $y = 8 \neq 0$

- Q: How can we change (c, d) while keeping the first two constraints binding at optimality? (i.e. we want the optimal solution to occur at the intersection of the lines $x + 2y = \mathrm{c}$ and $4x + 3y = \mathrm{d}$)

Changing Constraint Quantity

- Adjustment of y-intercepts of lines until lines don't intersect and one of the constraints is no longer "necessary"



- We can change $30 \leq c \leq 80$ and $60 \leq d \leq 160$

Ex: Beaver Creek Pottery

- Follow same steps in Excel from previous sensitivity analysis
- Sensitivity analysis for constraint quantities displayed below

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D6	Labor (hr/unit) Usage	40	16	40	40	10
D7	Clay (lb/unit) Usage	120	6	120	40	60

- If we fix $\mathrm{d} = 120\ lbs.\ of\ clay$, then $30 = 40 - 10 \leq c \leq 40 + 40 = 80$
- If we fix $\mathrm{c} = 40\ hours\ of\ labor$, then $60 = 120 - 60 \leq d \leq 120 + 40 = 160$

Shadow Prices

- The shadow price or dual value of a constraint (resource) correspond to the maximum amount that one would be willing to pay for one additional unit of that resource
- Standard sensitivity reports include these shadow prices
- In general, there is one shadow price for each constraint

Constraints

Cell	Name	Final Value	Shadow Price
D6	Labor (hr/unit) Usage	40	16
D7	Clay (lb/unit) Usage	120	6

Ex: Beaver Creek Pottery

- The shadow prices of labor and clay are 16 and 6, respectively

- Implication for labor
  - If we increase labor hours from 40 to $40 + x$, the profit increases by $16x$
  - We shouldn't pay more than $16 per hour of labor

- Implication for clay
  - If we increase pounds of clay from 120 to $120 + y$, the profit increases by $6y$
  - We shouldn't pay more than $6 per pound of clay

- This only can be applied for constraint quantity values in the limits under the specificity analysis

The End

Dale