

Lecture 6 T

Produced by Dr. Worldwide

- Quick-Screen is a clothing manufacturing company specializing in the production of commemorative shirts immediately following major sporting events and they have a contract to produce shirts for winning team of a college football bowl game on New Year's Day between State and Tech
- They will produce two different sweatshirts and two different t-shirts with one of each having writing on front (F) only and the other having writing on both front (F) and back (B)
- All items will be produced by the box where each box contains a dozen items
- Q: How much of each of the items should be produced to maximize profit?



- Decision Variables
 - $x_1 = Number \ of \ Boxes \ of \ Sweatshirts \ -F$
 - $x_2 = Number \ of \ Boxes \ of \ Sweatshirts B/F$
 - $x_3 = Number of Boxes of T shirts F$
 - $x_4 = Number \ of \ Boxes \ of \ T shirts B/F$
- Consider the following table showing resource requirements, unit costs, and profit of every dozen (box) of shirts

	Processing time (hr.) per dozen	Cost per dozen	Profit dozen
Sweatshirt - F	0.10	\$36	\$90
Sweatshirt - B/F	0.25	48	125
T-shirt - F	0.08	25	45
T-shirt - B/F	0.21	35	65



- Objective Function
 - Goal: Maximize profit on shirts
 - $Z = 90x_1 + 125x_2 + 45x_3 + 65x_4$
- Constraints
 - Only have 72 hours of processing time to produce all items: $0.1x_1 + 0.25x_2 + 0.08x_3 + 0.21x_4 \le 72$
 - Company has a budget of \$25,000: $36x_1 + 48x_2 + 25x_3 + 35x_4 \le 25,000$
 - Trailer truck will pick up shirts and can accommodate 1,200 standard-size boxes where each standard-size box holds 12 T-shirts and a box of 12 sweatshirts is 3 times the size of the standard-size box: 3(x₁ + x₂) + x₃ + x₄ ≤ 1,200
 - They have 500 dozens of blank sweatshirts: $x_1 + x_2 \le 500$
 - They have 500 dozens of blank T-shirts: $x_3 + x_4 \leq 500$
 - Nonnegativity: $x_1, x_2, x_3, x_4 \ge 0$



Download ProductMix.xlsx from website link called Sheet 1

• Before Excel solver

A product mix								
Products:	Sweatshirt-F	Sweatshirt-B/F	T-shirt-F	T-shirt-B/F				
	(dozen)	(dozen)	(dozen)	(dozen)				
Profit per dozen:	90	125	45	65				
Resources:					Usage	Constraint	Available	Left over
Processing time	0.1	0.25	0.08	0.21	0	<=	72	72
Cost	36	48	25	35	0	<=	25000	25000
Truck capacity	3	3	1	1	0	<=	1200	1200
Blank sweatshirts	1	1	0	0	0	<=	500	500
Blank T-shirts	0	0	1	1	0	<=	500	500
Production:								
Sweatshirts-F =	0							
Sweatshirts-B/F =	0							
T-shirt-F =	0							
T-shirt-B/F =	0							
Profit =	0							

• After Excel solver

A product mix								
Products:	Sweatshirt-F	Sweatshirt-B/F	T-shirt-F	T-shirt-B/F				
	(dozen)	(dozen)	(dozen)	(dozen)				
Profit per dozen:	90	125	45	65				
Resources:					Usage	Constraint	Available	Left over
Processing time	0.1	0.25	0.08	0.21	72	<=	72	0
Cost	36	48	25	35	21593.333	<=	25000	3406.6667
Truck capacity	3	3	1	1	1200	<=	1200	0
Blank sweatshirts	1	1	0	0	233.33333	<=	500	266.66667
Blank T-shirts	0	0	1	1	500	<=	500	0
Production:								
Sweatshirts-F =	175.55556							
Sweatshirts-B/F =	57.77778							
T-shirt-F =	500							
T-shirt-B/F =	0							
Profit =	45522.222							

- Recommended optimal solution to maximize profit at \$45,522.22
 - $x_1 = 175.56$
 - $x_2 = 57.78$
 - $x_3 = 500$
 - $x_4 = 0$
- Sensitivity report for objective function coefficients

Va	ariable	Cells						
			Final	Reduced		Objective	Allowable	Allowable
	Cell	Name	Value	Cost		Coefficient	Increase	Decrease
	\$B\$15	Sweatshirts-F = (dozen)	175.5555556		0	90	11.92307692	40
	\$B\$16	Sweatshirts-B/F = (dozen)	57.7777778		0	125	13.21428571	11.92307692
	\$B\$17	T-shirt-F = (dozen)	500		0	45	1E+30	4.111111111
	\$B\$18	T-shirt-B/F = (dozen)	0	-10.3333333	3	65	10.33333333	1E+30

• Sensitivity report for constraint quantities

Constrai	nts					
		Final	Shadow	Constraint	Allowable	Allowable
Cell	Name	Value	Price	R.H. Side	Increase	Decrease
\$F\$10) Blank sweatshirts Usage	233.3333333	0	500	1E+30	266.6666667
\$F\$11	Blank T-shirts Usage	500	4.111111111	500	185.7142857	500
\$F\$7	Processing time Usage	72	233.3333333	72	26.33333333	8.666666667
\$F\$8	Cost Usage	21593.33333	0	25000	1E+30	3406.666667
\$F\$9	Truck capacity Usage	1200	22.22222222	1200	260	316

- Linear program with 4 decision variables and 4 constraints requires more time to insert formulas in Excel
- Understanding of linear algebra can make this a more efficient process
- The object $\mathbf{x} = [x_1, x_2, x_3, x_4]$ is a row vector in \mathbb{R}^4

• The object
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$
 is a column vector in \mathbb{R}^4

• The transpose of a vector **x** , denoted **x**', transforms a row vector into a column vector and vice versa



• The object
$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$
 is a matrix in $\mathbb{R}^{4 \times 3}$

- The dimension of a matrix, denoted dim(A), describes its number of rows and number of columns (in that order)
- Based on above example, $\dim(\mathbf{A})$ is 4×3
- A row vector in \mathbb{R}^m is a matrix in $\mathbb{R}^{1 \times m}$
- A column vector in \mathbb{R}^n is a matrix in $\mathbb{R}^{n \times 1}$
- Typically, all vectors are by default column vectors



- For matrices $\mathbf{A} \in \mathbb{R}^{m \times n}$ and $\mathbf{B} \in \mathbb{R}^{n \times p}$, we can define their product $\mathbf{M} = \mathbf{AB}$, which will be a matrix in $\mathbb{R}^{m \times p}$
- For $\mathbf{A} \in \mathbb{R}^{3 \times 4}$ and $\mathbf{B} \in \mathbb{R}^{4 \times 2}$, matrix $\mathbf{M} = \mathbf{AB}$ can be expressed as

$$\mathbf{M} = \mathbf{A}\mathbf{B} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \end{bmatrix}$$
where

 $m_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{j3} + a_{i4}b_{4j}$ for i = 1,2,3 and j = 1,2

- In order to compute, $\mathbf{M}=\mathbf{AB},$ the number of columns in \mathbf{A} must equal the number of rows in \mathbf{B}
- In above example, the matrix $\mathbf{M} = \mathbf{B}\mathbf{A}$ does not exist



• If **A** and **B** are square matrices of the same size, say in $\mathbb{R}^{p \times p}$, we can compute both **AB** and **BA**, but they may not necessarily be equal

• Let
$$\mathbf{A} = \begin{bmatrix} 1 & 3 \\ -2 & 0 \end{bmatrix}$$
 and $\mathbf{B} = \begin{bmatrix} 2 & 5 \\ 7 & 1 \end{bmatrix}$
 $\mathbf{AB} = \begin{bmatrix} 1 \times 2 + 3 \times 7 & 1 \times 5 + 3 \times 1 \\ -2 \times 2 + 0 \times 7 & -2 \times 5 + 0 \times 1 \end{bmatrix} = \begin{bmatrix} 23 & 8 \\ -4 & -10 \end{bmatrix}$
 $\mathbf{BA} = \begin{bmatrix} 2 \times 1 + 5 \times -2 & 2 \times 3 + 5 \times 0 \\ 7 \times 1 + 1 \times -2 & 7 \times 3 + 1 \times 0 \end{bmatrix} = \begin{bmatrix} -8 & 6 \\ 5 & 21 \end{bmatrix}$

• Let
$$\mathbf{x} = [x_1, x_2, x_3, x_4]'$$
 and $\mathbf{y} = [y_1, y_2, y_3, y_4]'$ be column vectors in \mathbb{R}^4
 $\mathbf{x}'\mathbf{y} = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4 = \mathbf{y}'\mathbf{x}$



- MMULT Function in Excel
 - The MMULT Function in Excel is used to multiply arrays (matrices) that have compatible dimensions and returns an array (matrix)
 - Syntax: MMULT(array1,array2)
 - Example: Vector Multiplication

	А	В	С	D	E	F
1	Vector a	1	2	3		Vector b
2						3
3						2
4						1
5						
6	MMULT(a,b)	10				
7	MMULT(b,a)	3	6	9		
8		2	4	6		
9		1	2	3		

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	А	В	С	D	E	F
1	Vector a	1	2	3		Vector b
2						3
3						2
4						1
5						
6	MMULT(a,b)	10	=MMULT	(B1:D1,F2	:F4)	
7	MMULT(b,a)	3	6	9		
8		2	4	6		
9		1	2	3		

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1	Vector a	1	2	3		Vector b
2						3
3						2
4						1
5						
6	MMULT(a,b)	10				
7	MMULT(b,a)	3	6	9	=MMUL ⁻	T(F2:F4,
8		2	4	6		B1:D1)
9		1	2	3		

- MMULT Function in Excel
 - Example: Matrix Multiplication

	А	В	С	D	Е	F	G
1	Matrix A	1	2	3		Matrix B	
2		4	5	6		1	2
3						3	4
4							
5	MMULT(A,B)	#VALUE!					
6	MMULT(B,A)	9	12	15			
7		19	26	33			

- MMULT Function in Excel
 - Example: Matrix Multiplication

	А	В	С	D	E	F	G
1	Matrix A	1	2	3		Matrix B	
2		4	5	6		1	2
3						3	4
4							
5	MMULT(A,B)	#VALUE!	=MMULT	(B1:D2,F2	:G3)		
6	MMULT(B,A)	9	12	15			
7		19	26	33			

- MMULT Function in Excel
 - Example: Matrix Multiplication

	А	В	С	D	E	F	G	
1	Matrix A	1	2	3		Matrix B		
2		4	5	6		1		2
3						3		4
4								
5	MMULT(A,B)	#VALUE!						
6	MMULT(B,A)	9	12	15	=MMUL1	(F2:G3,B1	L:D2)	
7		19	26	33				



- SUMPRODUCT Function in Excel
 - The SUMPRODUCT function in Excel is used to multiply arrays (matrices) element-wise and then returns the sum of their products
 - In mathematics, this is often referred to as a dot-product or vector-product when the arrays are vectors
 - Syntax: SUMPRODUCT(array1,array2)
 - Ex: Usage on Vectors

	А	В	С	D	E	F
1	Vector a	1	2	3		Vector c
2	Vector b	3	2	1		4
3						5
4						6
5						
6	SUMPRODUCT(a,b)	10				
7	SUMPRODUCT(a,c)	#VALUE!				

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 - Syntax: SUMPRODUCT(array1,array2)
 - Ex: Usage on Vectors

	А	В	С	D	E	F
1	Vector a	1	2	3		Vector c
2	Vector b	3	2	1		4
3						5
4						6
5						
6	SUMPRODUCT(a,b)	10	=SUMPR	ODUCT(B	1:D1,B2:D)2)
7	SUMPRODUCT(a,c)	#VALUE!				

- SUMPRODUCT Function in Excel
 - The SUMPRODUCT function in Excel is used to multiply arrays (matrices) element-wise and then returns the sum of their products
 - In mathematics, this is often referred to as a dot-product when the arrays are vectors
 - Syntax: SUMPRODUCT(array1,array2)
 - Ex: Usage on Vectors

	А	В	С	D	E	F
1	Vector a	1	2	3		Vector c
2	Vector b	3	2	1		4
3						5
4						6
5						
6	SUMPRODUCT(a,b)	10				
7	SUMPRODUCT(a,c)	#VALUE!	=SUMPR	ODUCT(B	1:D1,F2:F	4)

- Both MMULT and SUMPRODUCT can be used in Excel to make the creation of formulas and constraints of linear programs considerably easier
- Made up example for practice

		_		_	-	_		-				
	A	В	C		D	E		F				
1	Profit	3	4							Solv	er	
2				Tot	al	Constraint	Max	x	Ľ			
3	Metal	2	1		3	<=		30	Г			
4	Plastic	0	4		4	<=		50				
5												
6					4	A B		С		D	E	F
7	Swag 1	1		1	Profi	t	3		4			
8	Swag 2	1		2						Total	Constraint	Max
9	Total	7		3	Meta	ıl 👘	2		1	30	<=	30
				4	Plast	ic	0		4	50	<=	50
				5								
				6				_				
				7	Swag	; 1	8.75					
				8	Swag	; 2	12.5					
				q	Tota	7	6 25					

• Creating EXCEL variable for easy referencing

Formulas Data Review	Viev	v Add-i	ns Help				
Lookup & Math & More Reference ~ Trig ~ Functions ~	Nam Manag		e Name 💉 Formula Y e from Selectic	on in the second s			
	7	Swag 1	1	New Nam	e	?	\times
	8	Swag 2	1	News			
	9	Total	7	<u>N</u> ame:	x		
	10			Scope:	Sheet4	×.	
	11			Comment:			
	12						
	13						
	14						
	15						
	16			Refers to:	=Sheet4!\$B\$7:\$B\$8		1
	17				ОК	Cancel	
	18						

• Usage of created variable in establishing constraints and objective function

	А	В	С	D	E	F
1	Profit	3	4			
2				Total	Constraint	Max
3	Metal	2	1	30	<=	30
4	Plastic	0	4	50	<=	50
5				=MMUL	T(B3:C4,x)	
6						
7	Swag 1	8.75				
8	Swag 2	12.5				
9	Total	76.25	=MMULT(B1:C1,X)		

• Another option for specification

SU	Μ	-	× ✓	$f_x = N$	/MULT(B3:C	4,B7:B8)
	А	В	С	D	E	F
1	Profit	3	4			
2				Total	Constraint	Max
3	Metal	2	1	B8)	<=	30
4	Plastic	0	4	4	<=	50
5						
6						
7	Swag 1	1				
8	Swag 2	1				
9	Total	7				

• Another option for specification

So	olver Parameters		
	Se <u>t</u> Objective:	\$B\$9	<u> </u>
	To: <u>Max</u> Mi <u>n</u> By Changing Variable Cells:	○ <u>V</u> alue Of:	0
	X Subject to the Constraints:	Add Constraint	×
	\$D\$3:\$D\$4 <= \$F\$3:\$F\$4	C <u>e</u> ll Reference:	Co <u>n</u> straint:
		\$D\$3:\$D\$4	<u>T</u> <=
		<u>О</u> К	<u>A</u> dd <u>C</u> ancel
	Ma <u>k</u> e Unconstrained Variables Non-	Negative	Load/Save
	S <u>e</u> lect a Solving Simplex LP Method:		✓ Options

- Try using MMULT and SUMPRODUCT in cell formulas
- Try using vectors/matrices in specification of constraints
- Download Lecture6WS.xlsx from course website from link Sheet 2 for all examples seen in this lecture









The End



Dale