
# Lecture 7

Produced by Dr. Worldwide

*Welcome to the 305*

# Ex: Investment


- Investor Kathy Allen has $70,000 to divide across multiple investments
  - Municipal bonds with 8.5% return
  - Certificate of deposit with 5% return
  - Treasury bills with 6.5% return
  - Growth stock with 13% return

- Q: How much should Kathy invest to maximize return?

- Guidelines for diversification
  - No more than 20% of the total investment should be in municipal bonds
  - Amount invested in CDs shouldn't exceed amount invested in the rest
  - At least 30% of the investment should be in treasury bills and CDs
  - More invested in CDs & treasury bills than in the other two by a ratio of at least 1.2 to 1
  - Kathy wants to invest the entire $70,000

# Ex: Investment

- Decision variables
  - $x_1 = Dollars\ invested\ in\ municipal\ bonds$
  - $x_2 = Dollars\ invested\ in\ CDs$
  - $x_3 = Dollars\ invested\ in\ Treasury\ Bills$
  - $x_4 = Dollars\ invested\ in\ Growth\ Stock$

- Linear program

Maximize $\quad 0.085x_1 + 0.05x_2 + 0.065x_3 + 0.13x_4$

Subject to

$$x_1 + x_2 + x_3 + x_4 = 70000$$
$$x_1/(x_1 + x_2 + x_3 + x_4) \leq 0.2$$
$$x_2 \leq x_1 + x_3 + x_4$$
$$(x_2 + x_3)/(x_1 + x_2 + x_3 + x_4) \geq 0.3$$
$$(x_2 + x_3)/(x_1 + x_4) \geq 1.2$$
$$x_1, x_2, x_3, x_4 \geq 0$$

# Ex: Investment


- Linear program in standard form

Maximize $\quad 0.085x_1 + 0.05x_2 + 0.065x_3 + 0.13x_4$

Subject to

$$
\begin{aligned}
& x_1 + x_2 + x_3 + x_4 = 70000 \\
& 0.8x_1 - 0.2x_2 - 0.2x_3 - 0.2x_4 \le 0 \\
& -x_1 + x_2 - x_3 - x_4 \le 0 \\
& 0.3x_1 - 0.7x_2 - 0.7x_3 + 0.3x_4 \le 0 \\
& 1.2x_1 - x_2 - x_3 + 1.2x_4 \le 0 \\
& x_1, x_2, x_3, x_4 \ge 0
\end{aligned}
$$

- Download Investment-1.xlsx from course website from link Sheet 1

- Optimal solution $(x_1, x_2, x_3, x_4) = (0,0,38181,3181.18)$

# Ex: Investment


- Sensitivity analysis

Variable Cells

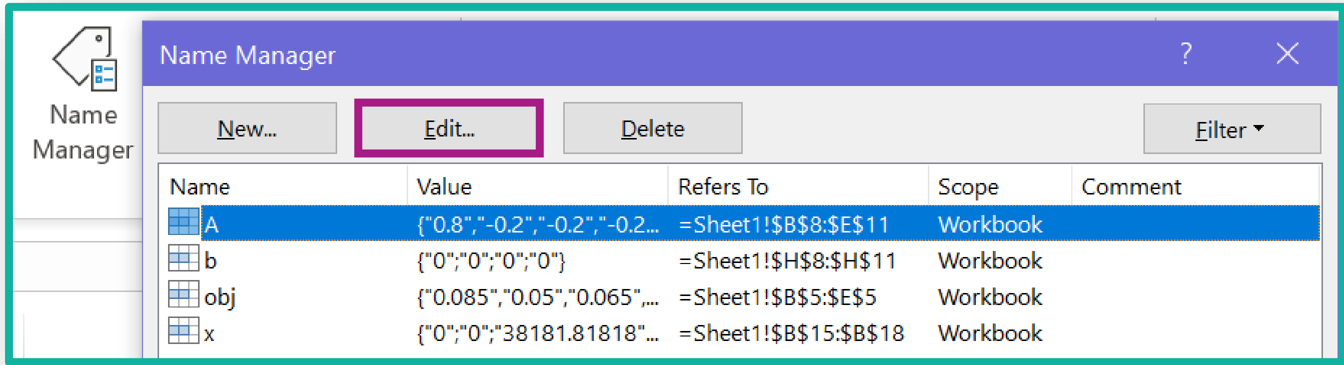
| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$15 | Municipal bonds = ($) | 0 | -0.045 | 0.085 | 0.045 | 1E+30 |
| $B$16 | CDs = ($) | 0 | -0.015 | 0.05 | 0.015 | 1E+30 |
| $B$17 | Treasury bills = ($) | 38181.81818 | 0 | 0.065 | 0.065 | 0.015 |
| $B$18 | Growth stock = ($) | 31818.18182 | 0 | 0.13 | 1E+30 | 0.045 |

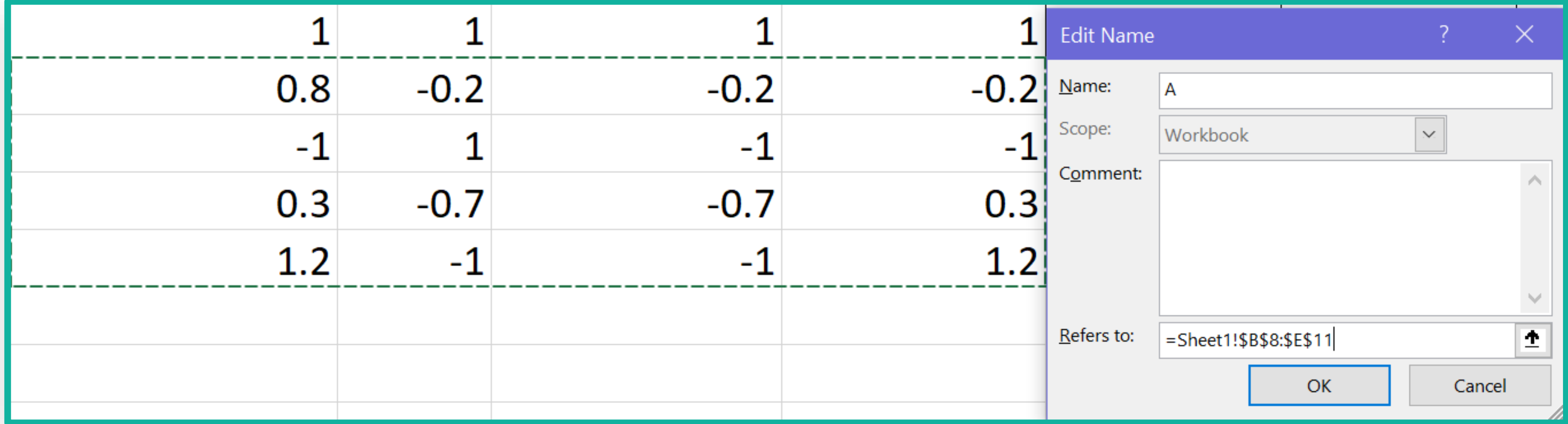
Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $F$7 | Total investment Usage | 70000 | 0.094545455 | 70000 | 1E+30 | 70000 |
| $F$8 | Constraint 1 Usage | -14000 | 0 | 0 | 1E+30 | 14000 |
| $F$9 | Constraint 2 Usage | -70000 | 0 | 0 | 1E+30 | 70000 |
| $F$10 | Constraint 3 Usage | -17181.81818 | 0 | 0 | 1E+30 | 17181.81818 |
| $F$11 | Constraint 4 Usage | 6.54836E-11 | 0.029545455 | 0 | 37800 | 70000 |

# Ex: Investment

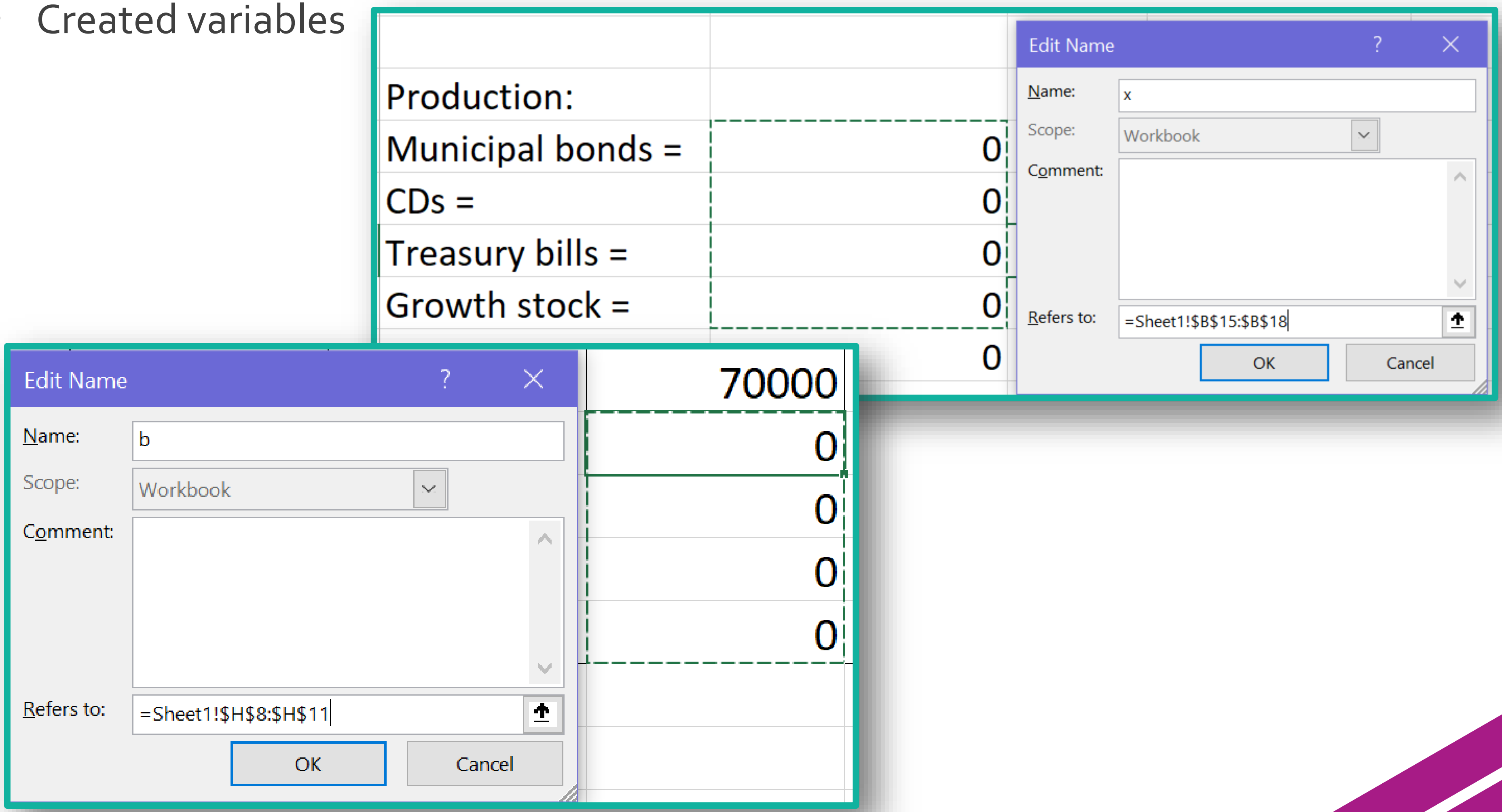
- Created variables




# Ex: Investment

- Created variables



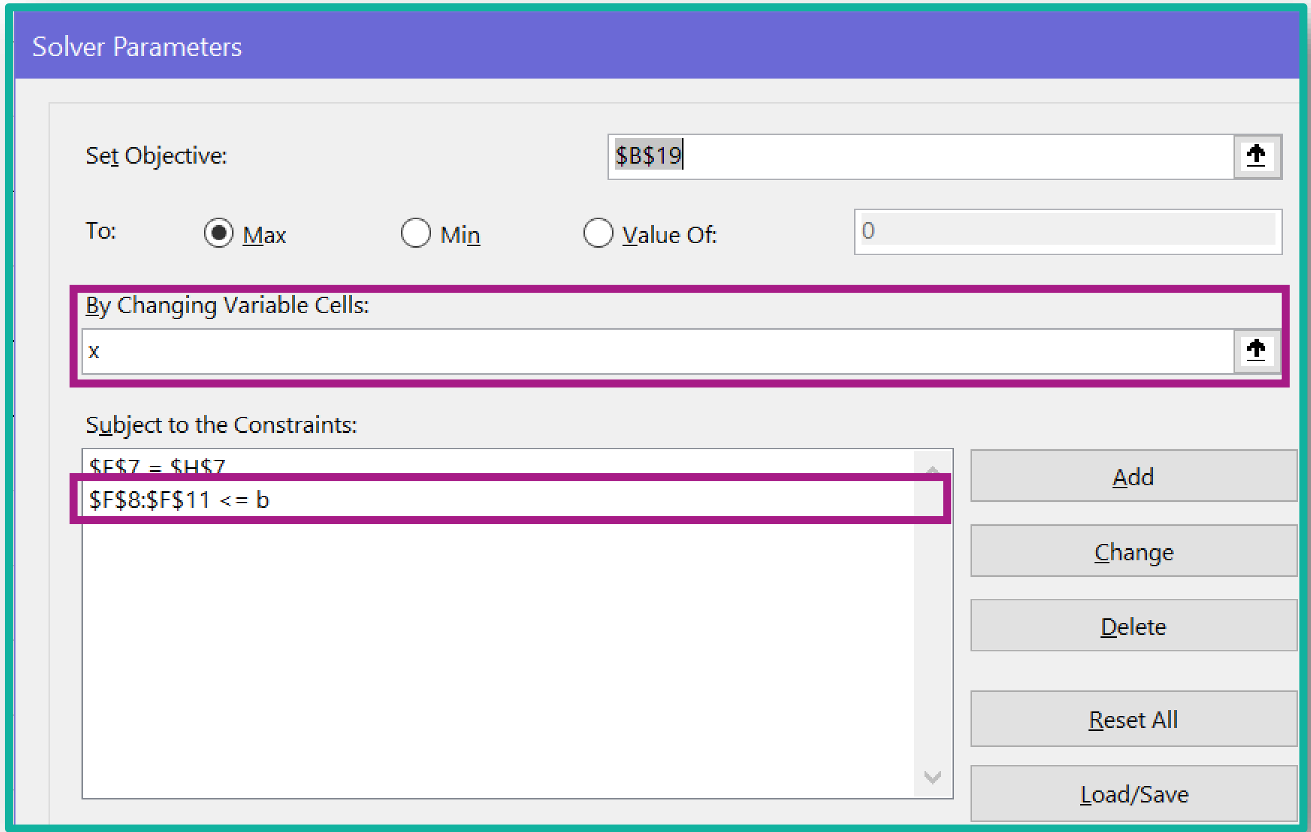
# Ex: Investment


- Usage of variables

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Products: | Municipal bonds ($) | CDs ($) | Treasury bills ($) | Growth stock ($) | | | |
| Return: | 0.085 | 0.05 | 0.065 | 0.13 | | | |
| Constraints: | | | | | Usage | Constraint | R.H.S. |
| Total investment | 1 | 1 | 1 | 1 | 0 | = | 70000 |
| Constraint 1 | 0.8 | -0.2 | -0.2 | -0.2 | =MMULT(A,x) | | 0 |
| Constraint 2 | -1 | 1 | -1 | -1 | 0 | <= | 0 |
| Constraint 3 | 0.3 | -0.7 | -0.7 | 0.3 | 0 | <= | 0 |
| Constraint 4 | 1.2 | -1 | -1 | 1.2 | 0 | <= | 0 |
| | | | | | | | |
| | | | | | | | |
| Production: | | | | | | | |
| Municipal bonds = | 0 | | | | | | |
| CDs = | 0 | | | | | | |
| Treasury bills = | 0 | | | | | | |
| Growth stock = | 0 | | | | | | |
| Return = | 0 | | | | | | |

# Ex: Investment

- Usage of variables




- Q: What other variable was created and how is it being used?

# Ex: Transportation


- Best Boy retail chain ships televisions from 3 of its distribution warehouses to three of its retail stores monthly
- Each warehouse has a fixed supply per month and fixed demand per month
- Q: How many TVs should be shipped from each warehouse to each store to minimize the total cost of transportation?
- Supply (700 TVs) and Demand (600 TVs)

| Warehouse | Supply (TVs) |
|---|---|
| 1. Cincinnati | 300 |
| 2. Atlanta | 200 |
| 3. Pittsburgh | 200 |

| Store | Demand (TVs) |
|---|---|
| A. New York | 150 |
| B. Dallas | 250 |
| C. Detroit | 200 |

# Ex: Transportation


- Shipping cost per TV for each route

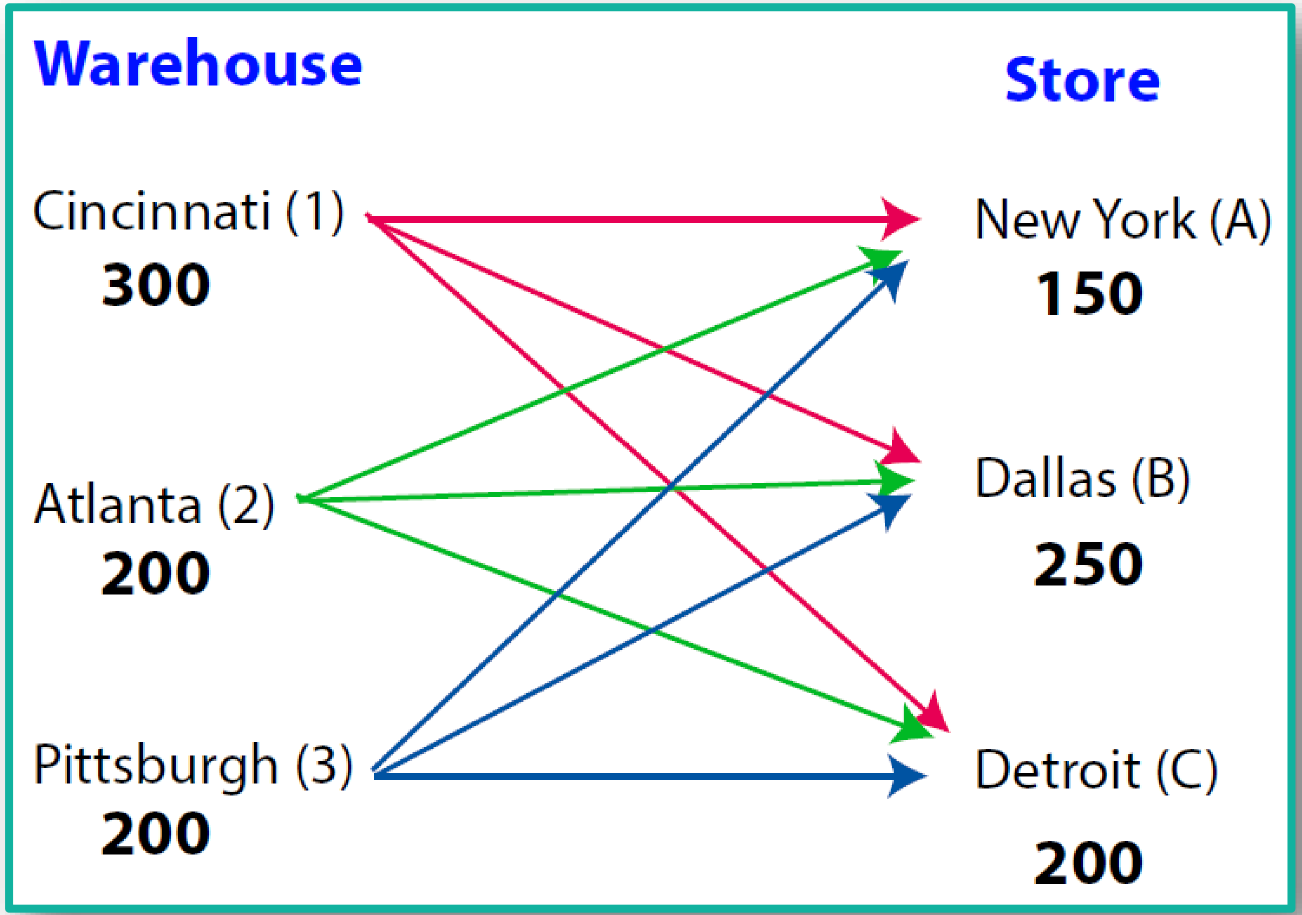
| Warehouse |
|---|
| 1. Cincinnati |
| 2. Atlanta |
| 3. Pittsburgh |

| Store |
|---|
| A. New York |
| B. Dallas |
| C. Detroit |

| | To Store | | |
|---|---|---|---|
| From Warehouse | A | B | C |
| 1 | $16 | $18 | $11 |
| 2 | 14 | 12 | 13 |
| 3 | 13 | 15 | 17 |

# Ex: Transportation

- Visual of all routes (supply > demand)



# Ex: Transportation


- Decision variables
  - Need to have one for each of the 9 routes
  - $x_{ij} = \text{number of televisions from warehouse } i \text{ to store } j$
  - $i = 1,2,3 \quad \& \quad j = A, B, C$

- Linear program in standard form

Minimize $\quad 16x_{1A} + 18x_{1B} + 11x_{1C} + 14x_{2A} + 12x_{2B} + 13x_{2C} + 13x_{3A} + 15x_{3B} + 17x_{3C}$

Subject to

$x_{1A} + x_{1B} + x_{1C} \leq 300$ (Cincinnati supply)

$x_{2A} + x_{2B} + x_{2C} \leq 200$ (Atlanta supply)

$x_{3A} + x_{3B} + x_{3C} \leq 200$ (Pittsburgh supply)

$x_{1A} + x_{2A} + x_{3A} \geq 150$ (New York demand)

$x_{1B} + x_{2B} + x_{3B} \geq 250$ (Dallas demand)

$x_{1C} + x_{2C} + x_{3C} \geq 200$ (Detroit demand)

# Ex: Transportation


- Download Transportation-1.xlsx from course website from link Sheet 2
- Sheet called Standard contains the standard linear program format and the sheet called Alternative contains a more compact form of the same linear program
- Focus on Alternative sheet

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 4 | ***Warehouse*** | New York | Dallas | Detroit | ***TV sets shipped*** | ***Constraint*** | ***Supply*** |
| 5 | Cincinatti | 0 | 0 | 200 | 200 | <= | 300 |
| 6 | Altanta | 0 | 200 | 0 | 200 | <= | 200 |
| 7 | Pittsburgh | 150 | 50 | 0 | 200 | <= | 200 |
| 8 | ***TV sets shipped*** | 150 | 250 | 200 | | | |
| 9 | ***Constraint*** | >= | >= | >= | | | |
| 10 | ***Demand*** | 150 | 250 | 200 | | | |
| 11 | ***Cost ($)*** | 7300 | | | | | |
| 12 | | | | | | | |
| 13 | | | | | | | |
| 14 | ***Warehouse*** | New York | Dallas | Detroit | | | |
| 15 | Cincinatti | 16 | 18 | 11 | | | |
| 16 | Altanta | 14 | 12 | 13 | | | |
| 17 | Pittsburgh | 13 | 15 | 17 | | | |

# Ex: Transportation


- Use of SUMPRODUCT in creation of objective function

| | A | B | C | D |
|---|---|---|---|---|
| 4 | ***Warehouse*** | New York | Dallas | Detroit |
| 5 | Cincinatti | 0 | 0 | 200 |
| 6 | Altanta | 0 | 200 | 0 |
| 7 | Pittsburgh | 150 | 50 | 0 |
| 8 | ***TV sets shipped*** | 150 | 250 | 200 |
| 9 | ***Constraint*** | >= | >= | >= |
| 10 | ***Demand*** | 150 | 250 | 200 |
| 11 | ***Cost ($)*** | 7300 | | |
| 12 | | =SUMPRODUCT(B5:D7, B15:D17) | | |
| 13 | | | | |
| 14 | ***Warehouse*** | New York | Dallas | Detroit |
| 15 | Cincinatti | 16 | 18 | 11 |
| 16 | Altanta | 14 | 12 | 13 |
| 17 | Pittsburgh | 13 | 15 | 17 |

# Ex: Transportation


- Searching for minimum of objective function

To: ○ Max ◉ Min ○ Value Of: 0

- Optimal solution
$(x_{1A}, x_{1B}, x_{1C}, x_{2A}, x_{2B}, x_{2C}, x_{3A}, x_{3B}, x_{3C}) = (0,0,200,0,200,0,150,50,0)$

- Textbook uses equality for demand instead of greater than or equal to

Subject to the Constraints:

`$B$8:$D$8 = $B$10:$D$10`

`$E$5:$E$7 <= $G$5:$G$7`

| | A | B | C | D |
|---|---|---|---|---|
| 2 | | | | |
| 3 | | | | |
| 4 | ***Warehouse*** | New York | Dallas | Detroit |
| 5 | Cincinatti | 0 | 0 | 200 |
| 6 | Altanta | 0 | 200 | 0 |
| 7 | Pittsburgh | 150 | 50 | 0 |
| 8 | ***TV sets shipped*** | 150 | 250 | 200 |
| 9 | ***Constraint*** | >= | >= | >= |
| 10 | ***Demand*** | 150 | 250 | 200 |

# Ex: Scheduling

- PM Computers assembles its own brand of laptops from component parts purchased overseas and domestically
- Most computers sold locally to the university, individuals, and businesses
- PM has production capacity to produce 160 computers per week with an additional 50 computers with overtime
- Cost per computer is $190 during regular time and $260 during overtime
- Additionally, it costs $10 per computer per week to hold a computer in inventory for future delivery
- PM wants to meet all customer orders with no shortages and quality service

# Ex: Scheduling

- Order schedule for the next 6 weeks

| Week | Computer Orders |
|---|---|
| 1 | 105 |
| 2 | 170 |
| 3 | 230 |
| 4 | 180 |
| 5 | 150 |
| 6 | 250 |

- Q: How much regular time and overtime production is needed each week to meet its orders at the minimum total production cost?

# Ex: Scheduling

- Each week, PM can produce computers either during regular time or during overtime.

- Each week, computers not used for an order are rolled over to the next week

- After the 6-week period, PM wants no inventory left over

- Decision variables
  - $r_j = regular\ production\ of\ computers\ per\ week\ j$
  - $o_j = overtime\ production\ of\ computers\ per\ week\ j$
  - $i_j = extra\ computers\ carried\ over\ as\ inventory\ in\ week\ j$
  - $j = 1,2,3,4,5,6$

# Ex: Scheduling

- Linear program in standard form

Minimize

$$190(r_1 + r_2 + r_3 + r_4 + r_5 + r_6)$$
$$+260(o_1 + o_2 + o_3 + o_4 + o_5 + o_6)$$
$$+10(i_1 + i_2 + i_3 + i_4 + i_5)$$

Subject to

$$r_1 + o_1 - i_1 = 105$$
$$r_2 + o_2 + i_1 - i_2 = 170$$
$$r_3 + o_3 + i_2 - i_3 = 230$$
$$r_4 + o_4 + i_3 - i_4 = 180$$
$$r_5 + o_5 + i_4 - i_5 = 150$$
$$r_6 + o_6 + i_5 = 250$$

$$r_i \leq 160 \; for \; i \in \{1,2,3,4,5,6\}$$
$$o_i \leq 50 \; for \; i \in \{1,2,3,4,5,6\}$$

$$r_i, o_i, i_i \geq 0 \; for \; i \in \{1,2,3,4,5,6\}$$

# Ex: Scheduling


- Download Multischedule-1.xlsx from course website from link Sheet 3
- Sheet called Standard contains the standard linear program format and the sheet called Alternative contains a more compact form of the same linear program
- From Standard, see the following solution from Excel Solver

| | Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | Week 6 |
|---|---|---|---|---|---|---|
| **Regular** | 160 | 160 | 160 | 160 | 160 | 160 |
| **Overtime** | 0 | 0 | 25 | 20 | 30 | 50 |
| **Inventory** | 55 | 45 | 0 | 0 | 40 | |
| ***Total*** | *160* | *215* | *230* | *180* | *190* | *250* |
| ***Required*** | *105* | *170* | *230* | *180* | *150* | *250* |

- Try to acquire the same solution from the Alternative format


# The End

Dale