Lecture 8

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Welcome to the 305

Integer Programming

- Prior linear programs have decision variables that are naturally integer-valued
- Optimal solutions are commonly not integer-valued
- Simply rounding up or down could lead to non-optimal solutions or could lie in an infeasible region
- Algorithms exist to handle this common problem
- Models where some/all the variables are required to be integer-valued are known as integer programming models

Integer Programming

- Total integer models are linear programming models where all the decision variables must be integer-valued
- 0-1 integer models are linear programming models where all the decision variables must take the values 0 or 1
- Mixed integer models are linear programming models where some of the decision variables must be integer valued while others do not

Ex: Total Integer Model

- Machine shop owner is planning to expand by purchasing some new machines – presses and lathes
- Owner estimated each press purchased will increase profit \$100 per day, and each lathe purchased will increase profit \$150 per day
- Number of machines the owner can purchase is limited by the cost of the machines and available floor space

Machine	Required Floor Space (ft.^{2})	Purchase Price
Press	15	\$8,000
Lathe	30	4,000

Ex: Total Integer Model

- Owner has a budget of $40,000 and 200 square feet of floor space
- Q: How many of each type of machine should be purchased to maximize the daily increase in profit?
- Decision variables
  - $x = number\ of\ presses$
  - $y = number\ of\ lathes$
- Linear program in standard form

Maximize $\quad 100x + 150y$

Subject to

$$8000x + 4000y \leq 40000$$
$$15x + 30y \leq 200$$
$$x, y \in \{0,1,2,\cdots\}$$

Ex: 0-1 Integer Model

- Community council must decide which recreation facilities to construct
  - Swimming pool
  - Tennis center
  - Athletic field
  - Gymnasium

- Q: Which facilities should be constructed to maximize daily usage?

- Council's decision is subject to land and cost limitations

Recreation facility	Expected usage (people/day)	Cost	Land requirements (acres)
Swimming pool	300	35000	4
Tennis center	90	10000	2
Athletic field	400	25000	7
Gymnasium	150	90000	3

Ex: 0-1 Integer Model

- Community has $120,000 budget and 12 acres of land
- Swimming pool and tennis center cannot both be constructed
- Binary decision variables (indicator variables)
  - $x_1 = \begin{cases} 1 & if\ swimming\ pool\ is\ constructed \\ 0 & otherwise \end{cases}$
  - $x_2 = \begin{cases} 1 & if\ tennis\ center\ is\ constructed \\ 0 & otherwise \end{cases}$
  - $x_3 = \begin{cases} 1 & if\ athletic\ field\ is\ constructed \\ 0 & otherwise \end{cases}$
  - $x_4 = \begin{cases} 1 & if\ gymnasium\ is\ constructed \\ 0 & otherwise \end{cases}$

Ex: 0-1 Integer Model

- Linear program in standard form

Maximize $300x_1 + 90x_2 + 400x_3 + 150x_4$

Subject to

$$35000x_1 + 10000x_2 + 25000x_3 + 90000x_4 \leq 120000$$
$$4x_1 + 2x_2 + 7x_3 + 3x_4 \leq 12$$
$$x_1 + x_2 \leq 1$$
$$x_1, x_2, x_3, x_4 \in \{0,1\}$$

- The constraint $x_1 + x_2 \leq 1$ is known as a mutually exclusive constraint which reflects the contingency that either the swimming pool or tennis center can be constructed but not both

Ex: 0-1 Integer Model

- Q: What restriction would the constraint $x_1 + x_2 = 1$ imply?
- Q: What restriction would the constraint $x_1 + x_2 + x_4 = 2$ imply?
- The previous two constraints are also known as multiple-choice constraints
- Commonly, multiple-choice questions can have a single answer, but sometimes multiple-choice questions may have multiple answers
- Q: What restriction would the constraint $x_1 + x_2 + x_3 + x_4 \geq 2$ imply?
- Q: What restriction would the constraint $x_1 + x_2 + x_3 + x_4 \leq 3$ imply?
- These previous two constraints are variations of the multiple-choice constraint

Ex: 0-1 Integer Model

- Q: What restriction regarding the swimming pool (x_1) and the tennis center (x_2) would the constraint $x_2 \leq x_1$ imply?

- The constraint $x_2 \leq x_1$ is known as a conditional constraint and implies that one facility being constructed is conditional on another building be constructed?

- Q: What restriction would the constraint $x_2 = x_1$ imply?

- When the inequality is replaced with an equal sign in a conditional constraint, we get a corequisite constraint?

Ex: Mixed Integer Model

- Nancy Smith has $250,000 to invest in 3 alternative investments for the year
  - Condominiums cost $50,000 and return $9,000 in 1 year
  - Land costs $12,000 per acre and will return $1,500 per acre in 1 year
  - Municipal bonds cost $8,000 and will return $1,000 in 1 year
- Only 4 condominiums, 15 acres of land, and 20 municipal bonds available
- Q: How many condominiums, acres of land, and municipal bonds should be purchased to maximize return?
- Q: How is this an example of a mixed integer model?

Ex: Machine Shop

- Linear program in standard form

Maximize	$100x + 150y$	(Profit)
Subject to	$8000x + 4000y \leq 40000$	(Cost)
	$15x + 30y \leq 200$	(Floor Space)
	$x, y \in \{0,1,2,\cdots\}$	(Integer Requirement)

- Simplified linear program in standard form

Maximize	$100x + 150y$
Subject to	$2x + y \leq 10$
	$3x + 6y \leq 40$
	$x, y \in \{0,1,2,\cdots\}$

Ex: Machine Shop

- Download MachineShop.xlsx from course website from link Sheet 1
- Q:What is the problem with the optimal solution?

	A	B	C	D	E	F	G
1	**Machine Shop**						
2							
3	Products:	Presses	Lathes				
4		(unit)	(unit)				
5	Increase in profit per unit:	100	150				
6	Resources:			Usage	Constraint	Available	Left over
7	Budget	2	1	10	<=	10	0
8	Space	3	6	40	<=	40	0
9							
10							
11	Production:						
12	Presses	2.2222222					
13	Lathes	5.5555556					
14	Profit =	1055.5556					

Ex: Machine Shop

- We could fix the problem by rounding to the nearest integer

  $(2.222, 5.556) \to (2,6)$

- The problem is the point may be infeasible

  $3(2) + 6(6) = 42 \geq 40$

- We could fix the problem by rounding down

  $(2.222, 5.556) \to (2,5)$

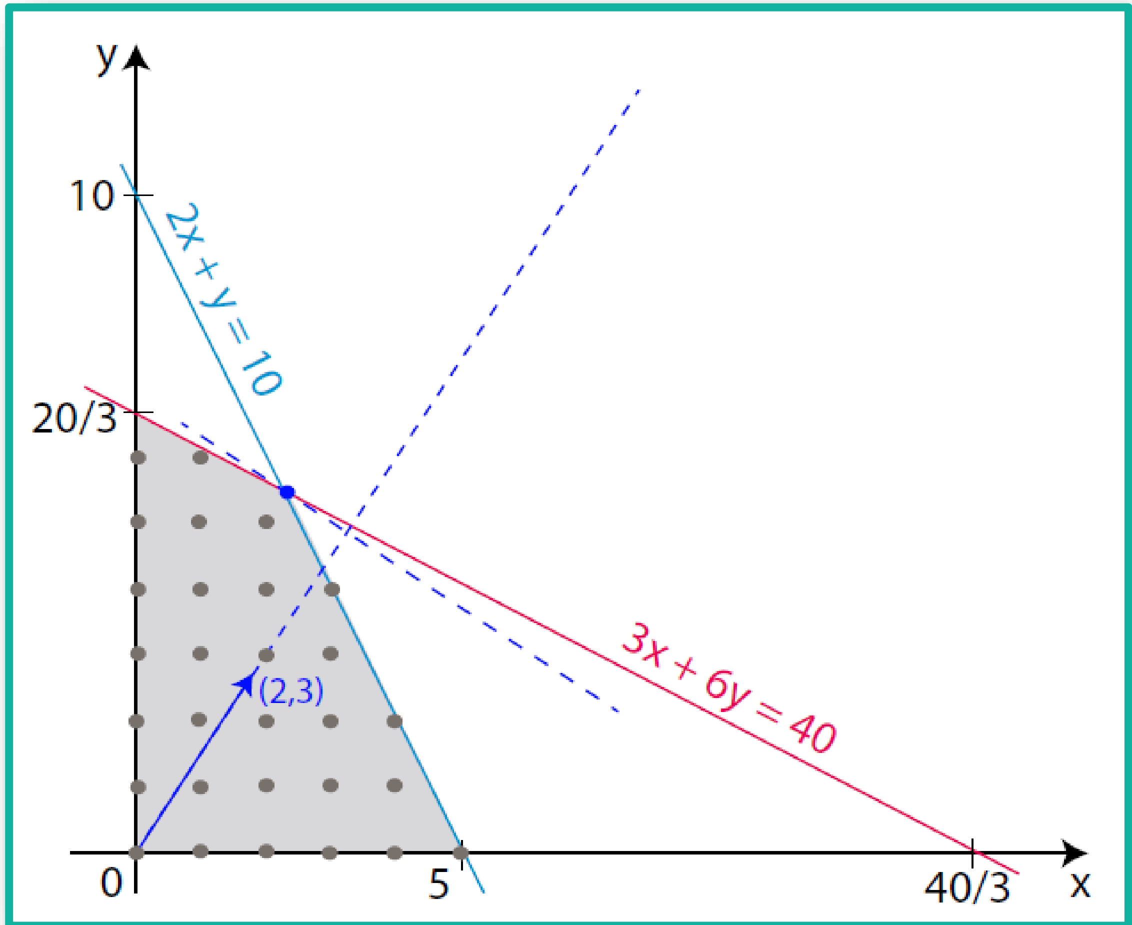
- This point is feasible but may not be optimal

  $100(2) + 150(5) = \$950$

- No clear rule on how to round the solution to acquire the optimal solution for the integer programming model

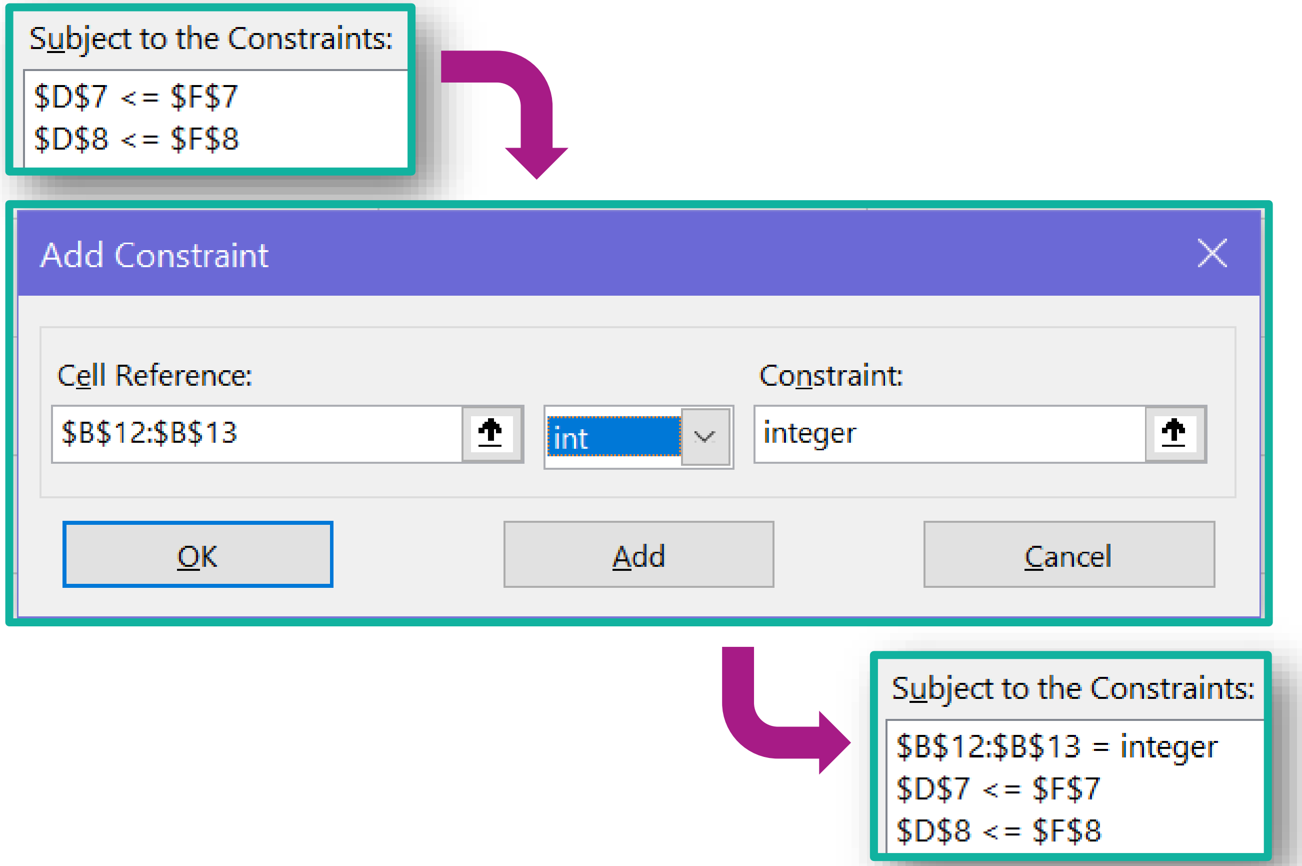
Ex: Machine Shop

- Graphical solution



Ex: Machine Shop

- Finding optimal integer solution from Excel Solver



Ex: Machine Shop

- Optimal solution to integer programming model

	A	B	C	D	E	F	G
1	**Machine Shop**						
2							
3	Products:	Presses	Lathes				
4		(unit)	(unit)				
5	Increase in profit per unit:	100	150				
6	Resources:			Usage	Constraint	Available	Left over
7	Budget	2	1	8	<=	10	2
8	Space	3	6	39	<=	40	1
9							
10							
11	Production:						
12	Presses	1					
13	Lathes	6					
14	Profit =	1000					

The End

Dale