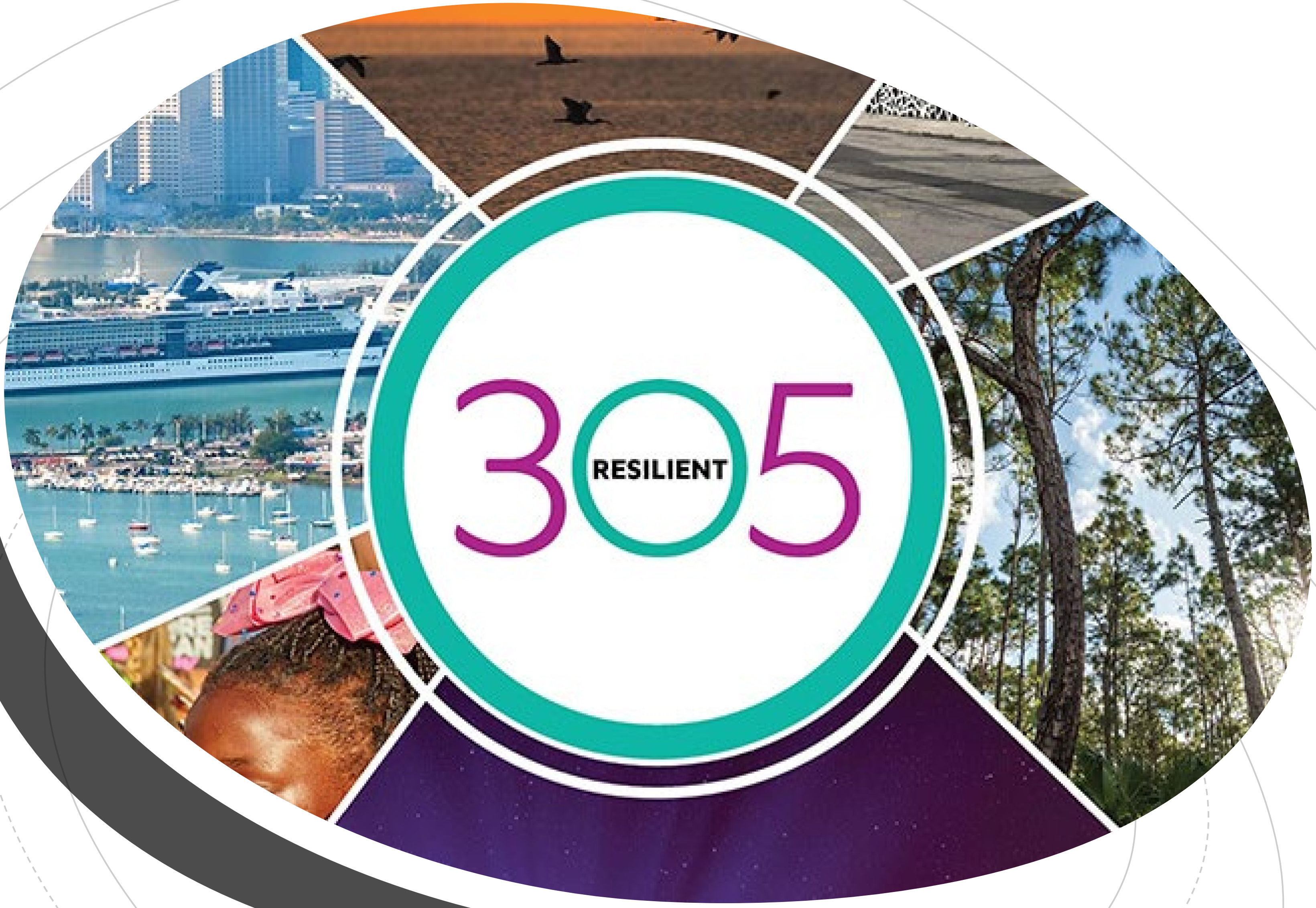
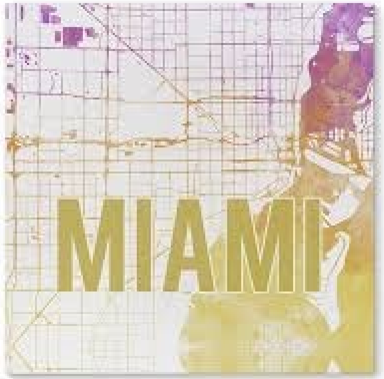


Lecture 9

Produced by Dr. Worldwide

Welcome to the 305



Ex: Capital Budgeting

- University bookstore is considering several expansion projects
- Some projects require 2-years and some projects require 3 years

Project	NPV return ($1,000s)	Project costs/year ($1,000s) 1	2	3
1. Website	$120	$55	$40	$25
2. Warehouse	85	45	35	20
3. Clothing department	105	60	25	-
4. Computer department	140	50	35	30
5. ATMs	70	30	30	-
Available funds per year		$150	$110	$60

- Not enough space available for computer and clothing department

Ex: Capital Budgeting

- Q: Which projects should the director select to maximize returns?

- Binary decision variables (indicator variables)
  - $x_1 = \begin{cases} 1 & \text{if website selected} \\ 0 & \text{otherwise} \end{cases}$
  - $x_2 = \begin{cases} 1 & \text{if warehouse selected} \\ 0 & \text{otherwise} \end{cases}$
  - $x_3 = \begin{cases} 1 & \text{if clothing department selected} \\ 0 & \text{otherwise} \end{cases}$
  - $x_4 = \begin{cases} 1 & \text{if computer department selected} \\ 0 & \text{otherwise} \end{cases}$
  - $x_5 = \begin{cases} 1 & \text{if ATM selected} \\ 0 & \text{otherwise} \end{cases}$

Ex: Capital Budgeting

- Linear program in standard form

Maximize $120x_1 + 85x_2 + 105x_3 + 140x_4 + 70x_5$

Subject to

$$55x_1 + 45x_2 + 60x_3 + 50x_4 + 30x_5 \leq 150$$
$$40x_1 + 35x_2 + 25x_3 + 35x_4 + 30x_5 \leq 110$$
$$25x_1 + 20x_2 + 30x_4 \leq 60$$
$$x_3 + x_4 \leq 1$$
$$x_1, x_2, x_3, x_4 \in \{0,1\}$$

- Decision variables are binary making this a 0-1 Integer Model

- Download CapitalBudgeting.xlsx from course website from link Sheet 1

Ex: Capital Budgeting

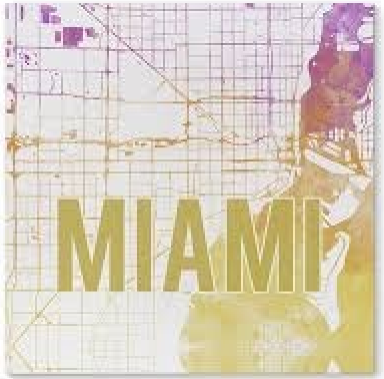
- Modifying constraints for binary decision variables

	A	B	C	D	E	F	G	H	I
1	**Capital budgeting**								
2									
3	Projects to pursue	Website	Warehouse	Clothing Dept.	Computer Dept.	ATMs			
4									
5	NPV return ($1000s):	120	85	105	140	70			
6	Resources:						Spent	Constraint	Available
7	Budget year 1	55	45	60	50	30	135	<=	150
8	Budget year 2	40	35	25	35	30	105	<=	110
9	Budget year 3	25	20	0	30	0	55	<=	60
10	Space constraint	0	0	1	1	0	1	<=	1
11									
12	Indicators of selected projects:								
13	Website	1							
14	Warehouse	0							
15	Clothing Dept.	0							
16	Computer Dept.	1							
17	ATMs	1							
18	NPV return =	330							

Subject to the Constraints:

$B$13:$B$17 = binary
$G$10 <= $I$10
$G$7 <= $I$7
$G$8 <= $I$8
$G$9 <= $I$9

Ex: Set Covering



- American Parcel Service (APS) has determined it needs to add several new package distribution hubs to service cities east of the Mississippi River

- APS desires to construct the minimum set of new hubs in the following 12 cities so that there is a hub within 300 miles of each city

City	Cities within 300 miles
1. Atlanta	Atlanta, Charlote, Nashville
2. Boston	Boston, New York
3. Charlotte	Atlanta, Charlotte, Richmond
4. Cincinnati	Cincinnati, Detroit, Indianapolis, Nashville, Pittsburgh
5. Detroit	Cincinnati, Detroit, Indianapolis, Milwaukee, Pittsburgh
6. Indianapolis	Cincinnati, Detroit, Indianapolis, Milwaukee, Nashville, St. Louis
7. Milwaukee	Detroit, Indianapolis, Milwaukee
8. Nashville	Atlanta, Cincinnati, Indianapolis, Nashville, St. Louis
9. New York	Boston, New York, Richmond
10. Pittsburgh	Cincinatti, Detroit, Pittsburgh, Richmond
11. Richmond	Charlotte, New York, Pittsburgh, Richmond
12. St. Louis	Indianapolis, Nashville, St. Louis

Ex: Set Covering

- Binary decision variables (indicator variables)
  - $x_i = \begin{cases} 1 & \text{if city } i \text{ is selected to be a hub} \\ 0 & \text{otherwise} \end{cases}$
  - $i \in \{1, 2, \cdots, 12\}$

- Q: How can we select the minimum number of hubs that cover all the cities?

- Objective function

$$Z = x_1 + x_2 + \cdots + x_{12} = \sum_{i=1}^{12} x_i$$

- We need to specify individual constraints for all 12 cities because we need to cover all 12 cities

Ex: Set Covering

- Constraints to ensure covering of first 3 cities

City	Cities within 300 miles
1. Atlanta	Atlanta, Charlote, Nashville
2. Boston	Boston, New York
3. Charlotte	Atlanta, Charlotte, Richmond

$x_1 + x_3 + x_8 \geq 1$ (To Cover Atlanta)

$x_2 + x_9 \geq 1$ (To Cover Boston)

$x_1 + x_3 + x_{11} \geq 1$ (To Cover Charlotte)

- Download SetCovering.xlsx from course website from link Sheet 2

- Run Excel Solver to find the solution

City
1. Atlanta
2. Boston
3. Charlotte
4. Cincinnati
5. Detroit
6. Indianapolis
7. Milwaukee
8. Nashville
9. New York
10. Pittsburgh
11. Richmond
12. St. Louis

Ex: Set Covering

- Minimum number of distribution hubs needed is 4
  - Optimal solution given below with duplicate cities underlined

1.	Atlanta	Atlanta, Charlote, Nashville
2.	Boston	Boston, New York
6.	Indianapolis	Cincinnati, Detroit, Indianapolis, Milwaukee, Nashville, St. Louis
11.	Richmond	Charlotte, New York, Pittsburgh, Richmond

  - Other optimum solutions exist i.e. {Boston, Charlotte, Detroit, St. Louis}

Ex: Transporting Grain

- General transportation problem
  - There are sources and destinations
  - Sources have supply and destinations have demand
  - A cost is associated to transport units along each route

- Wheat is harvested in the Midwest and stored in grain elevators in 3 different cities
  - Kansas City
  - Omaha
  - Des Moines

- These grain elevators supply flour mills in 3 different cities
  - Chicago
  - St. Louis
  - Cincinnati

Ex: Transporting Grain

- Supply and demand each month in tons

Grain Elevator	Supply
1. Kansas City	150
2. Omaha	175
3. Des Moines	275
Total	600 tons

Mill	Demand
A. Chicago	200
B. St. Louis	100
C. Cincinnati	300
Total	600 tons

- Grain is shipped in railroad cars, each capable of holding 1 ton of wheat

- Transportation cost per ton (railroad car) of wheat

	Mill		
Grain elevator	**A. Chicago**	**B. St. Louis**	**C. Cincinnati**
1. Kansas City	$6	$8	$10
2. Omaha	7	11	11
3. Des Moines	4	5	12

Ex: Transporting Grain

- Download Mills.xlsx from course website from link Sheet 3
- Q: How many tons of wheat should be shipped on each route to minimize cost?
- Decision variables
  - x_{ij} = number of tons of grain to ship from i to j
  - $i \in \{1,2,3\}$
  - $j \in \{A, B, C\}$
- Objective function

$$Z = 6x_{1A} + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} + 4x_{3A} + 5x_{3B} + 12x_{3C}$$

Ex: Transporting Grain

- Constraints

$x_{1A} + x_{1B} + x_{1C} = 150$ (Kansas City Supply)
$x_{2A} + x_{2B} + x_{2C} = 175$ (Omaha Supply)
$x_{3A} + x_{3B} + x_{3C} = 275$ (Des Moines Supply)

$x_{1A} + x_{2A} + x_{3A} = 200$ (Chicago Demand)
$x_{1B} + x_{2B} + x_{3B} = 100$ (St. Louis Demand)
$x_{1C} + x_{2C} + x_{3C} = 300$ (Cincinnati Demand)

$x_{ij} \in \{0,1,2,\cdots\}$

- Optimal solution

Variables	**Destinations**				
Sources	A. Chicago	B. St. Louis	C. Cincinnati	Grain shippe	***Supply***
1. Kansas City	25	0	125	150	150
2. Omaha	0	0	175	175	175
3. Des Moines	175	100	0	275	275
Grain shipped	200	100	300		
Demand	200	100	300		
Total cost =	4525				

Ex: Transporting Grain

- Balanced versus unbalanced
  - When total supply equals total demand, the problem is balanced
  - When total supply doesn't equal total demand, the problem is unbalanced
  - Current grain transportation problem is balanced (600 Supply = 600 Demand)

- Modifications for unbalanced transportation problems
  - If total supply is smaller than total demand, we replace the equalities in the demand constraints to be $\leq$
  - If total supply is bigger than total demand, we replace the equalities in the supply constraints to be $\leq$
  - Alternative approach is to create slack variables to absorb excess
  - When total supply is smaller than total demand, the slack variables act as fictitious sources
  - When total supply is bigger than total demand, the slack variables act as fictitious destinations

Ex: Transporting Grain

- Suppose we change Cinicinnati's demand from 300 to 350

- New constraints

$x_{1A} + x_{1B} + x_{1C} = 150$ (Kansas City Supply)
$x_{2A} + x_{2B} + x_{2C} = 175$ (Omaha Supply)
$x_{3A} + x_{3B} + x_{3C} = 275$ (Des Moines Supply)

$x_{1A} + x_{2A} + x_{3A} \leq 200$ (Chicago Demand)
$x_{1B} + x_{2B} + x_{3B} \leq 100$ (St. Louis Demand)
$x_{1C} + x_{2C} + x_{3C} \leq 350$ (Cincinnati Demand)

$x_{ij} \in \{0,1,2,\cdots\}$

- Total supply (600) is smaller than total demand (650)

Ex: Transporting Grain

- Include fictitious source "Slack (S)"

- Modified constraints with slack variable

$x_{1A} + x_{1B} + x_{1C} = 150$ (Kansas City Supply)
$x_{2A} + x_{2B} + x_{2C} = 175$ (Omaha Supply)
$x_{3A} + x_{3B} + x_{3C} = 275$ (Des Moines Supply)
$x_{SA} + x_{SB} + x_{SC} = 50$ (Fictitious "Slack" Supply)

$x_{1A} + x_{2A} + x_{3A} + x_{SA} = 200$ (Chicago Demand)
$x_{1B} + x_{2B} + x_{3B} + x_{SB} = 100$ (St. Louis Demand)
$x_{1C} + x_{2C} + x_{3C} + x_{SC} = 350$ (Cincinnati Demand)

$x_{ij} \in \{0,1,2,\cdots\}$

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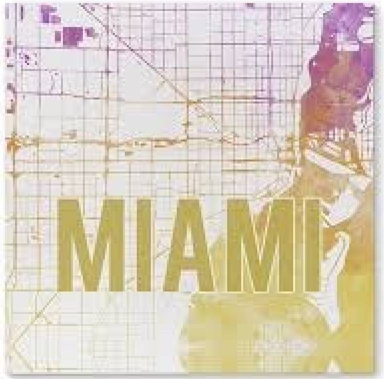
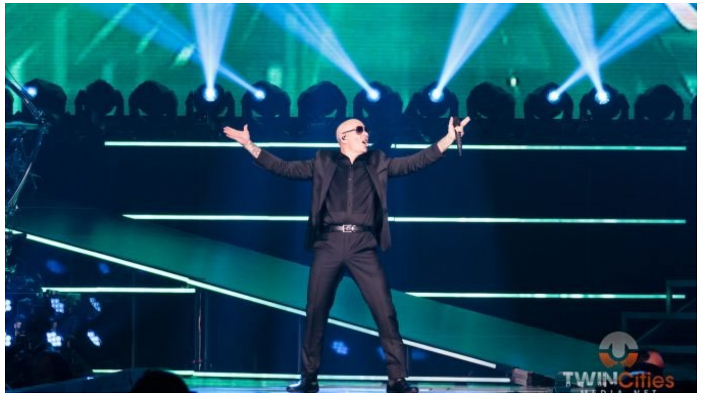
Ex: Transporting Grain

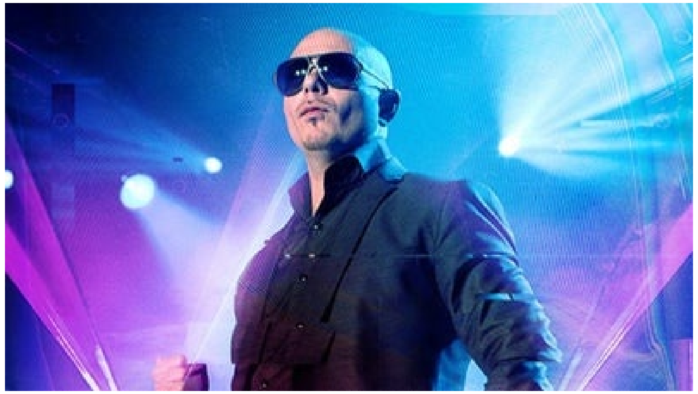


- Optimal solution from Solver

Variables	**Destinations**				
Sources	A. Chicago	B. St. Louis	C. Cincinnati	Grain shipped	***Supply***
1. Kansas City	0	0	150	150	150
2. Omaha	25	0	150	175	175
3. Des Moines	175	100	0	275	275
4. Slack source	0	0	50	50	50
Grain shipped	200	100	350		
Demand	200	100	350		
Total cost =	4525				

- Q: Why is all the grain from the slack variables for Cincinnati?





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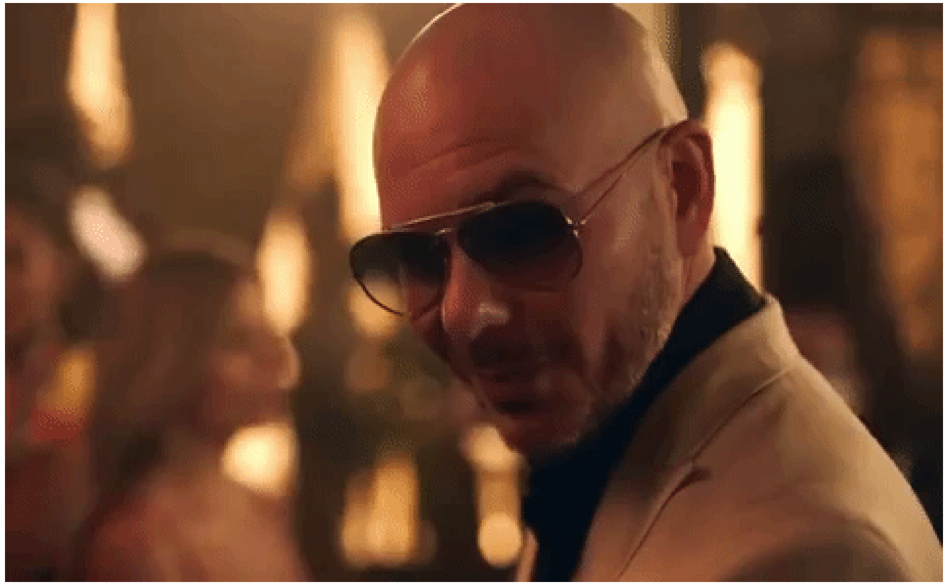

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