An Article Submitted to

Journal of Quantitative Analysis in Sports

Manuscript 1030

Optimal End-Game Strategy in Basketball

David H. Annis*

*Naval Postgraduate School, annis@nps.edu

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Abstract

When faced with protecting a three-point lead in the waning seconds of a basketball game, which is a preferable strategy: playing defense or fouling the offense before they can attempt a game-tying shot? Gonzaga University head coach, Mark Few, was faced with such a decision against Michigan State in the semi-finals of the Maui Invitational (November 22, 2005) and elected to play defense. The strategy backfired, as Michigan State's Maurice Ager made a three-point basket at the buzzer to force overtime. (Gonzaga eventually won in triple overtime.) Was this failure to hold the lead at the end of regulation bad luck or bad strategy? Put another way, which strategy (conventional defense or intentionally fouling) maximizes the defensive team's chances of winning the game? Drawing on the Gonzaga/Michigan State game for inspiration, this paper addresses this question and concludes that, contrary to popular belief, intentionally fouling is preferable to playing tight defense.

KEYWORDS: strategy, basketball, optimal, intentional foul

1 Introduction

On November 22, 2005, Michigan State and Gonzaga played a triple-overtime thriller in the semi-finals of the Maui Invitational. Although Gonzaga eventually won, they needed the extra periods because Michigan State's Maurice Ager sank a game-tying three-point basket at the end of regulation. The next morning, the Seattle Times [4] began its recap by rhetorically asking its readers,

> The debate is one of college basketball's most spirited: Leading by three points in the final seconds, do you foul before a shooter can fire in a [three-point field goal] that ties the game?

Gonzaga coach Mark Few answered the question during a post-game interview. (The transcript appears in the Seattle Times article [4]. Few's comments were in response to an on-air question posed by ESPN college basketball analyst Jay Bilas, who comes to the same conclusion as Few [1].)

> It's not a debate for me ... If the guy [fouled] hits the free throw, they can tip it out and hit a three and you can lose the game. They can score four on you ... I've never, ever fouled, and I've been in that situation a lot of times. I think you've got to make somebody hit a real tough shot. And they did.

Gonzaga University President Robert J. Spitzer called Few "a gifted strategist" [2]. But in this situation, did his strategy measure up to his reputation? Under similar circumstances (leading by three points with one defensive possession remaining in regulation or an overtime period), what is the optimal strategy for protecting the lead?

2 Mathematical Modeling

2.1 Assumptions

For the purposes of discussion, we assume that there are only two possible strategies which a coach can employ in these situation. The first strategy is to play conventional defense, and the second is to commit an intentional, non-shooting foul before the opponent can attempt a potentially game-tying shot. Given coach Few's opinion on the matter, we refer to the two options as the Few and non-Few strategies, respectively.

To compare these strategies, some assumptions must be made about possible events in the waning seconds of regulation and in overtime (should it be necessary):

A1 The team protecting the three-point lead will not score again in regulation.

A2 The team trailing will not attempt a two-point basket when trailing by three.

A3 The team trailing will get at most one offensive rebound.

A4 The team trailing is in the double bonus (i.e., if fouled while not in the act of shooting, they will be awarded two foul shots).

A5 A team trailing by more than one point with one free throw remaining will intentionally miss that free throw.

A6 If the offensive team trails by less than three points (e.g., after a made free throw) and secures an offensive rebound, they will attempt a tip-in rather than a three-point basket.

A7 If the game goes to overtime, each team has a 50% chance of winning.

Assumptions **A1** and **A2** make explicit our focus on games with one defensive possession remaining. In this situation, the team protecting a lead will not have the opportunity to score again. Furthermore since this is the last possession, it makes no sense for the offensive team to settle for two points, which ensures a loss. Assumption **A3** is realistic and provides an explicit end to computation. It is freely conceded that the complexity of the problem is greater than what is described here. However, no depth of detail can be considered exhaustive, and this description is a reasonable approximation. Because the offensive team is attempting to win the game (rather than lose by a smaller margin), they must employ the strategies dictated by **A4** and **A5**. Failure to do so guarantees a loss and is therefore unreasonable.

Assumption **A6** is the most interesting. Recall, coach Few's concern with intentionally fouling is the potential for a made free-throw, an offensive rebound and a made three-point basket, resulting in a loss for his team in regulation.[1] The offense's strategy assumed by **A6** explicitly precludes Few's worst fears, however it is his opponent's optimal strategy in virtually all circumstances. This can be seen by comparing the offense's probability of winning the game in each case. Should the trailing team attempt a tip-in or put-back following an offensive rebound, they will win with probability $0.5 \times p_{\mathrm{TI}}$, where

[1]His preoccupation with this possibility is interesting, since it is also possible to lose in regulation if, in the course of playing defense, a shooter hits a three-point basket, is fouled and makes the subsequent free throw.

p_{TI} is the probability of successfully converting a tip-in and the conditional probability of winning in overtime is one-half. Conversely, if they attempt a desperation three-point basket, their probability of winning is p_{D3}, where p_{D3} is the probability of a made basket under those conditions. So the offensive team is better off attempting the tip-in, provided $p_{\mathrm{TI}} \geq 2p_{\mathrm{D3}}$, which is practically assured. Even in if $p_{\mathrm{TI}} < 2p_{\mathrm{D3}}$, some coaches may instruct their teams to attempt a put-back, as this strategy minimizes their chance of losing in regulation (assuming that the probability of a tip-in exceeds that of a desperation three-point heave). Finally, **A7** addresses the probability of winning in overtime. This is the only logical assumption as half the teams which play in overtime must win and half must lose.

2.2 Analysis

Because many things can happen in the course of a single defensive possession, the non-Few strategy is simpler to analyze than the Few strategy. The probability of winning if the coach employs the preemptive fouling (non-Few) strategy can be assessed using a simple decision tree, given in Figure 1. Arrows are labeled with the corresponding event names, and terminal nodes (corresponding to overtime or a win for the defensive team) are given in all capital letters.

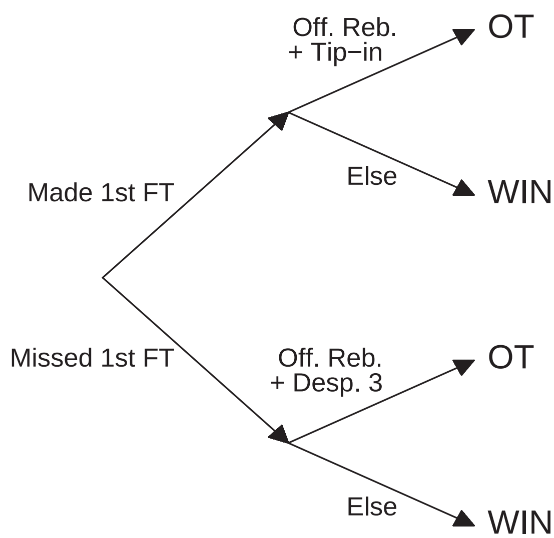

Figure 1: Decision tree for outcomes of the non-Few strategy.

Because the non-Few strategy entails fouling the offensive player before a shot can be attempted, the possible game conclusions all involve the initial free throw attempt. Regardless of whether the first free throw is converted, the second must be missed intentionally. The outcome of the first free throw only influences the offensive team's strategy should they rebound the missed free throw (see **A6**). Figure 2 displays the probabilities associated with the events in Figure 1. Note that the terminal nodes (WIN and OT) have been replaced by 1 and 1/2, respectively, to denote the probability of the defensive team winning in those situations.

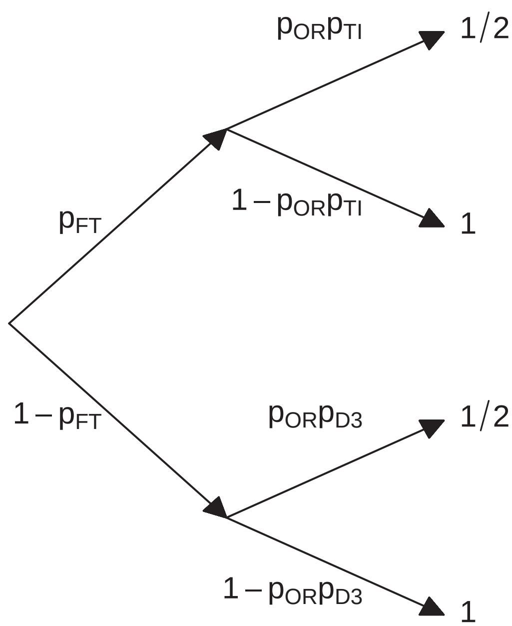

Figure 2: Decision tree for outcomes of the non-Few strategy.

Based on Figure 2, the defensive team's probability of winning the game under the non-Few strategy, p_{nF}, is

$$p_{\mathrm{nF}} = p_{\mathrm{FT}}\left[\frac{1}{2}p_{\mathrm{OR}}p_{\mathrm{TI}} + (1 - p_{\mathrm{OR}}p_{\mathrm{TI}})\right] + (1 - p_{\mathrm{FT}})\left[\frac{1}{2}p_{\mathrm{OR}}p_{\mathrm{D3}} + (1 - p_{\mathrm{OR}}p_{\mathrm{D3}})\right], \quad (1)$$

where p_{FT} is the probability of a made free throw, p_{OR} is the probability of an offensive rebound after a missed free throw, p_{TI} is the probability of converting a subsequent tip-in and p_{D3} is the probability of converting a desperation three-point basket.

If, on the other hand, a coach decides to play defense (i.e., the Few strategy), the probability of his team eventually winning can be determined by the decision tree in Figure 3. However, rather than enumerate the possible outcomes associated with winning and losing the games, it is more convenient to bound the probability of winning for comparison with p_{nF}.

Consider the first level of the tree in Figure 3. If the team trailing makes a three-point basket (denoted by MADE 3), the best case scenario for the

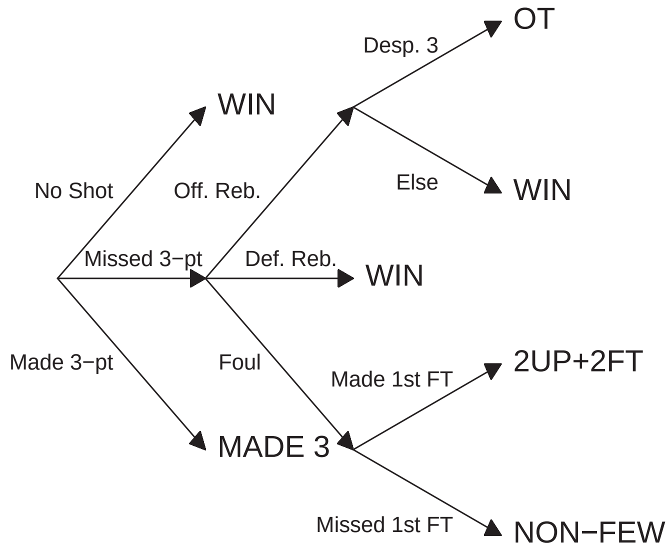

Figure 3: Decision tree for outcomes of the Few strategy.

defending team is overtime. (If the shooter is fouled, he can win the game for the offense in regulation by hitting the free throw.) Similarly, the terminal node labeled 2UP+2FT corresponds to the situation in which the defensive team is up by two points with two free throws for the offense forthcoming. Clearly this is a less desirable situation for the defense than had they employed the non-Few strategy and fouled immediately (in which case the would lead by three – not two – with two free throws to come).

Therefore the probability of winning using the Few strategy, p_{F}, is bounded from above by p^*_{F}, which is calculated from the best-case (for the defense) tree in Figure 4. Probabilities associated with possible outcomes of the Few strategy are given in Figure 5, from which similar calculations yield the probability of victory for the defensive team.

$$p_{\mathrm{F}} \leq p_{\mathrm{NS}} + \frac{1}{2}p_{\mathrm{3pt}} + (1 - p_{\mathrm{NS}} - p_{\mathrm{3pt}})\times \left[p_{\mathrm{OR}}\left(\frac{1}{2}p_{\mathrm{D3}} + (1 - p_{\mathrm{D3}})\right) + p_{\mathrm{DR}} + (1 - p_{\mathrm{OR}}p_{\mathrm{DR}})p_{\mathrm{nF}}\right], \qquad (2)$$

where p_{NS} is the probability that the offense attempts no shot, p_{3pt} is the probability of the offensive team converting a three-point attempt and p_{DR} is the probability of a defensive rebound after a missed shot. The other terms (p_{nF}, p_{FT}, p_{OR}, p_{TI} and p_{D3}) are defined in Equation 1. Unfortunately, it's not obvious whether or not $p_{nF} > p_F$ or vice versa. However, the two can be compared given estimates of the relevant probabilities. In order to evaluate p_{nF} in Equation 1, assume:

$$p_{\mathrm{FT}} = 0.75 \qquad p_{\mathrm{OR}} = 0.15 \qquad p_{\mathrm{TI}} = 0.7 \qquad p_{\mathrm{D3}} = 0.1,$$

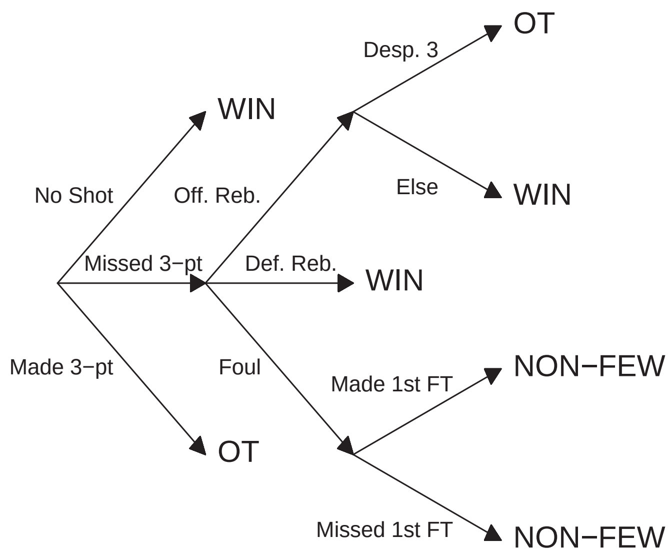

Figure 4: Upper bound of decision tree for outcomes of the Few strategy.

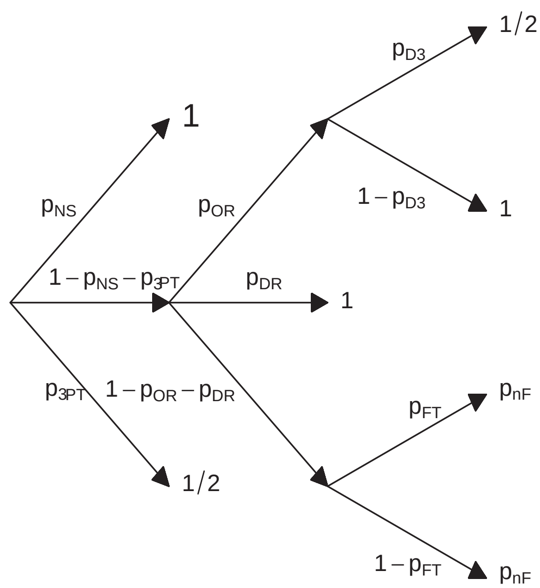

Figure 5: Upper bound of decision tree for outcomes of the Few strategy.

which results in $p_{\text{nF}} = 0.9588$. Similarly, to evaluate p_{F} in Equation 1, assume: assume $p_{\text{NS}} = 0.1$, $p_{\text{DR}} = 0.7$ and $p_{\text{3pt}} = 0.25$. Under these circumstances, $p_{\text{F}} = 0.8661$. These computations suggest two things. First, immediately fouling the offensive team is a superior strategy; and second, this is a problem many coaches envy, as even the sub-optimal strategy results in winning in excess of 85% of the time.

3 Discussion

In order to conduct the analysis a number of assumptions were made. It is reasonable, therefore, to ask to what extent the conclusion (namely, the non-Few strategy is superior) depends on these assumptions. First, consider that the probability of winning by playing defense (the Few strategy) was replaced by its upper bound. In other words, this simplification explicitly favors the Few strategy and represents a best-case scenario for electing to play defense. So if the non-Few (foul immediately) strategy still looks appealing even considering these pro-Few assumptions, it must be better.

The model presented captures many of the salient features of the problem, however, by no means is this characterization exhaustive. On the other hand, this analysis illustrates a general framework for resolving similar, more detailed strategic questions. While we feel that the present analysis is a suitable compromise between intricacy and tractability, in principle the decision trees may be extended to any level of complexity desired.

Conceding the approximate nature of this analysis, we proceed to sensitivity of the conclusion on assumed probabilities used to evaluate Equations 1 and 2. For many plausible assumed probabilities, $p_{\mathrm{nF}} > p_{\mathrm{F}}$. (The appendix contains a brief snippet of R code [3] which can be used to evaluate the expressions in Equations 1 and 2.) We find that *for virtually any reasonable values of these probabilities*, intentionally fouling the opponent increases the chances of eventually winning the game.

Gonzaga head coach Mark Few and ESPN college basketball analyst Jay Bilas prefer protecting a slim lead late by playing tight defense rather than intentionally fouling. However, analysis shows that for a wide range of assumptions, this strategy is less desirable than fouling immediately.

Appendix

```
compare.probs <- function(p.FT=0.75, p.OR=0.15, p.TI=0.9, p.D3=0.1,
                          p.NS=0.1, p.DR=0.7, p.3pt=0.25) {

  ###########################################################
  # Compare probability of winning the game when leading by #
  # 3 points without the ball with one defensive possession #
  # remaining.                                              #
  # ------------------------------------------------------- #
  # p.FT = probability of made free throw                   #
  # p.OR = probability of offensive rebound on missed shot  #
  # p.TI = probability of tip-in or put-back after off reb  #
  # p.D3 = probability of made desperation 3-point FG       #
  # p.NS = probability of no shot by offense                #
  # p.DR = probability of defensive rebound on missed shot  #
  # p.3pt = probability of made 3-point FG                  #
  ###########################################################

  if (p.OR + p.DR > 1) stop ("p.OR + p.DR must not exceed 1")

  p.Foul <- 1-p.OR-p.DR

  # ------------------------------------------------------- #
  # Compute Win Probability under Non-Few Strategy EQN (1)  #
  # ------------------------------------------------------- #

  p.nF <- p.FT * ( 0.5*p.OR*p.TI + (1-p.OR*p.TI) ) +
          (1-p.FT) * ( 0.5*p.OR*p.D3 + (1-p.OR*p.D3) )

  # ------------------------------------------------------- #
  # Compute Upper Bound on Prob under Few Strategy EQN (X)  #
  # ------------------------------------------------------- #

  p.F <- p.NS + 0.5*p.3pt + (1-p.NS-p.3pt) *
           ( p.OR*(0.5*p.D3 + (1-p.D3)) + p.DR + p.Foul*p.nF )

  strat <- "Foul"
  if(p.F > p.nF) strat <- "Defense"

  return( list(Pr.Non.Few=p.nF, Pr.Few.MAX=p.F, Optimal.Strategy=strat) )

}
```

References

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